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Existence and global asymptotic stability of positive almost periodic solution for a predator-prey system in an artificial lake

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ABSTRACT

A periodic predator-prey model has been introduced in [5] to study the effect of water level on persistence or extinction of fish populations living in an artificial lake. By using the continuation theorem of Mawhin's coincidence degree theory, the authors give sufficient conditions for the existence of at least one positive periodic solution. In this paper we study the problem in the general case. We begin by analyzing the invariance, permanence, non-persistence and the globally asymptotic stability for the system. Most interestingly, under additional conditions, we find that the periodic solution obtained in [5] is unique. Finally, in order to make the model system more realistic, we consider the special case when the periodicity in [5] is replaced by almost periodicity. We obtain conditions for existence, uniqueness and stability of a positive almost periodic solution. The methods used in this paper will be comparison theorems and Lyapunov functions. An example is employed to illustrate our result.

1. Introduction

The dynamical relationship between predators and their prey has long been of the dominant themes in ecology [2,10,11,15–17,27]. Generally, all prey-predator models are derived from the following model system:

$$\begin{cases} \dot{x} = f(x) - g(x, y)y, \\ \dot{y} = eg(x, y)y - dy. \end{cases} \tag{1}$$

where x and y denote prey and predator densities at time t, respectively, $f(x)$ stands for the prey growth rate in the absence of predators, $g(x, y)$ denotes the average feeding rate of a predator (i.e., the functional response of predators to prey density). Parameters e and d denote the efficiency of predators to convert the consumed prey into predator's new offspring and predator mortality rate, respectively.

The functional response is usually assumed to increase with prey density, and decrease (or not change) with predator density, this can be classified as: (a) prey dependent, when prey density alone determines the response; (b) predator dependent, when both predator and prey populations affect the response, and (c) ratio dependent when the feeding rate is determined by the ratio of prey density to predator density. A historical account of biological relevance of functional response is available in [27].

Naturally, environmental periodicity and fluctuations have great influence on the interaction between prey and predator species, these characteristics lead us to propose a change in the prey-predator model (1) aiming at the inclusion of the environmental fluctuations in this system of differential equations. There are some literature where authors have considered the non-autonomous ordinary differential equation models to study the models with seasonally varying parameters (see [4,6,9,12,25,28,31,32] and the references therein). More and more researchers begin to investigate ecological systems with random perturbation subjected to environmental noise, for example, in [7,8], the authors show that the random fluctuations play a crucial role in population dynamics which can affect significantly the time behavior of prey-predator systems (see also [29,30]).

Recently, in [5], the authors proposed a new functional response in order to explain the influence of changing water levels fluctuations in the Pareloup lake on predator-prey interactions. The Pareloup lake situated in the south of France is one of the major Hydro Electric Projects in the Country. In the proposed model, the authors used the population of Roach species (Gardon in French) as prey and the Pike species (Brochet in French) as predator. Pike and Roach are the most important species in this lake. This functional response is based on the following general considerations.

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When a predator attacks a prey, it has access to a certain quantity of food depending on the water level. When the water level is low, during the autumn, the predator is more in contact with the prey, and the predation increases. Conversely, when the water level is high, in the spring, it is more difficult for the predator to find a prey and the predation decreases. It is assumed that the accessibility function $r(t)$ for the prey is continuous and 1-periodic, the minimum value r_1 is reached in spring and the maximum value r_2 is attained during autumn. The predator needs a quantity γ_B as food, but it has access to a quantity

$$g(x,y) = \frac{r(t)x}{y+D},$$

where D measures the other causes of mortality outside the predation. If

$$g(x,y) = \frac{r(t)x}{y+D} \geq \gamma_B,$$

then the predator will be satisfied with the quantity γ for his food. Otherwise, that is, if

$$g(x,y) = \frac{r(t)x}{y+D} \leq \gamma_B,$$

the predator will content himself with

$$g(x,y) = \frac{r(t)x}{y+D}.$$

Consequently, the quantity of food received by one predator is

$$\min\left(\frac{r(t)x}{y+D}, \gamma_B\right).$$

The authors in [5] studied the following non-autonomous predator-prey model

$$\begin{cases} \dot{G}(t) = \gamma_G G(t) - m_G G^2(t) - \min\left(\dfrac{r(t)G(t)}{B(t)+D}, \gamma_B\right)B(t), \\ \dot{B}(t) = e_B \min\left(\dfrac{r(t)G(t)}{B(t)+D}, \gamma_B\right)B(t) - m_B B(t), \\ G(0) = G_0 > 0, \quad B(0) = B_0 > 0, \end{cases} \tag{2}$$

where $G(t)$ and $B(t)$ represent the densities of the prey and the predator, respectively, at time t. m_G and m_B are, respectively, the consumption rate of biomass by metabolism of prey and predator. γ_G and γ_B denote the maximum consumption rate of resource by the prey and predator, respectively. e_B is the conversion rate. By using Gaines and Mawhin's continuation theorem of coincidence degree theory [20], the authors have established sufficient conditions for the existence of positive periodic solutions of the predator-prey system (2). Such a solution describes an equilibrium situation consistent with the variability of environmental conditions and such that both populations survive. The trajectories in the phase plane of these solutions of the non-autonomous system take the place of the equilibria points of the autonomous system. see [5,24,26] for more details). The existence of periodic solutions and their stability for a delayed version of system (2) are studied in [22]. In [23] the authors study the predator-prey model (2) with harvesting terms. They show that the conditions that guarantee the multiple positive periodic solutions depend on the harvesting terms. In order to incorporate the mechanism of diffusion into the populations model, the author in [24] proposed a reaction-diffusion model to study the effect of diffusion and water level on the persistence of the two specifically fish populations. Through the proposed models, it is possible to verify that the change in the water level is directly associated with the variation of the number of fish species present in the lake.

In a more general case, when we consider the effects of environmental factors, almost periodicity is sometimes more realistic and more general than periodicity. Because there is no a priori reason to expect the existence of periodic solutions. We assume here that the predation rate is almost periodic function. We obtain sufficient conditions for the existence of a unique globally attractive positive almost periodic solution of system (2).

The contents of the paper are as follows. In Section 2, we study in details the dynamics of the general non-autonomous case of (2) and establish sufficient conditions for the boundedness, permanence, predator extinction, and globally asymptotic stability. Under some additional conditions, we conclude that the periodic solution obtained in [5] is unique and is globally asymptotically stable. Section 3 is for the case when the predation rate is almost periodic. We provide sufficient conditions for the existence and globally asymptotic stability of a unique positive almost periodic solution of system (2). We end up by simulation results and concluding remarks.

2. General case

In this section, we assume that $r(t)$ is continuous and bounded above and below by positive constants r_1 and r_2 respectively. We shall explore the dynamics of the non-autonomous predator-prey system (2) and present some results including the positive invariance, permanence, predator extinction and the global asymptotic stability.

2.1. Positive invariance and permanence

For many biological systems, boundedness of solutions and permanence are important. They give biological sense of system. We first show that system (2) is well-posed in the sense that for any positive initial conditions (G_0, B_0), there exists a unique solution of the system (2), which remains positive and bounded, and hence exists globally. To this end, we establish the following result.

Let $h: (t, G, B) \to \min(f(t, G, B), \gamma_B)$.

Lemma 1. *If f is locally lipschitz, then the function h is locally lipschitz.*

Proof. It is easy to see that

$$\min\left(f(t,G,B), \gamma_B\right) = \frac{f(t,G,B) + \gamma_B - |f(t,G,B) - \gamma_B|}{2}.$$

The form of h with respect to f obviously shows that if f is locally lipschitz, then h is locally lipschitz. Hence, local existence and uniqueness properties are obtained for the corresponding Cauchy problem [14]. □

The state space of (2) remains in the positive octant $\mathbb{R}^2_+ = \{(G,B) : G \geq 0, B \geq 0\}$. Indeed, the set $\mathbb{R}^2_+$ is positively invariant since the vector field of (2) is inward on the boundary of $\partial\mathbb{R}^2_+$.

Now, we shall prove the permanence of system (2). We first rewrite the system (2) in a simpler form. We suppose that:

$$r_2 < \min\left(\frac{\gamma_B(B_0+D)}{G_0}, \frac{4m_G\gamma_B m_B D}{(\gamma_G+m_B)^2}\right). \tag{H1}$$