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Volumetric behavior quantification to characterize trajectory in phase space

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ABSTRACT

This paper presents a methodology to extract a number of quantifier features to characterize volumetric behavior of trajectories in phase space. These features quantify expanding and contracting behaviors and complexity that can be used in nonlinear and chaotic signals classification or clustering problems. One of the features is directly extracted from the distance matrix and seven features are extracted from a matrix that is subsequently obtained from the distance matrix. To illustrate the proposed quantifiers, Mackey–Glass time series and Lorenz system were employed and feature evaluation was performed. It is shown that the proposed quantifier features are robust to different initializations and can quantify volumetric behavior characteristics. In addition, the ability of these features to differentiate between signals with different parameters is compared with some common nonlinear features such as fractal dimensions and recurrence quantification analysis features.

1. Introduction

There are two separate, but interacting lines of development characterizing chaos and nonlinear theory. The first line focuses on ordinary nonlinear differences and differential equations that may have chaotic behavior meaning the system is available. In the second line, the system is not available and relies heavily on the computational study of chaotic system outputs and includes methods for investigating potential chaotic behavior in observed time series.

Describing global and local behavior of trajectories can lead to a better understanding of attractor properties. These properties of attractor can give us valuable information about systems and their behavior. For example, Lyapunov exponents that are extracted from trajectory can indicate dissipation of the system [1]. In this paper, eight features based on local and global behaviors of trajectory in phase space are proposed in terms of volumetric and complexity. Lyapunov exponents provide rate of local separation in each dimension of space, while the proposed method can provide a single value of expansion rate for the whole trajectory globally. Moreover, the rates of expansion and contraction will be achieved separately. Fractal dimensions focus on occupying space capacity in detail [2], whereas the proposed method presents a feature that provides occupied space globally. The complexity feature in the proposed method presents a new meaning of complexity that has a different meaning from approximate [3] and sample [4] entropies. This meaning has a relationship with the variations in expansion and contraction speed. Some of the proposed features have independent meanings and some other features have meanings in comparison to other features. These features quantify some properties of the trajectories obtained from nonlinear and chaotic signals. Therefore, they can be employed in classification problems in applications such as biomedical signal processing, finance, electronics, etc, in which the observed signals are nonlinear or chaotic.

The rest of the paper is organized as follows. Section 2 reviews some related works. The proposed method is described in Section 3. Section 4 is devoted to evaluate and discuss the proposed method by comparing two nonlinear systems with different parameters. Finally, our conclusions are stated in Section 5.

2. Related work

In many studies, trajectory in phase space is reconstructed from time series and features or properties are extracted. These features characterize the behavior of trajectories or attractors that help to identify or classify systems and trace their changes. For example,

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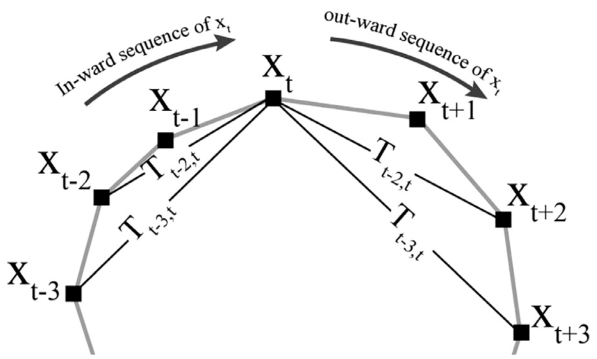


Fig. 1. Trajectory in phase space. In-ward and out-ward sequences of $\mathbf{x}_t$ are shown. $\mathbf{T}_{i,j}$ is distance between $\mathbf{x}_i$ and $\mathbf{x}_j$.

there are entropy-based features [5] such as approximate entropy (ApEn) [3], which is a technique to quantify the amount of regularity and unpredictability of fluctuations over time-series data [6], and sample entropy (SampEn), which is a modification of approximate entropy, used extensively for assessing complexity of a physiological time-series signal, thereby diagnosing diseased state [4]. Lyapunov exponent is a quantifier that characterizes the rate of separation of infinitesimally close trajectories [1,7]. The characteristics of some features are focused on measuring the space-filling capacity of patterns that illustrate how a fractal scales differently from the space it is embedded in [8], namely Fractal dimension [2] such as Higuchi [9]. Katz feature [10] characterizes stretching and distribution of trajectory in phase space by comparing the relationship between the length of trajectory and diagonals. In some cases, quantification of behavior of signals or systems is done by a transform such as Discrete Fourier Transform (DTF) [11], Discrete Wavelet Transform(DWT) [12], and Singular Value Decomposition (SVD) [13]. These transforms are relatively general and can be used in a variety of applications. Local Fractional z-Transforms [14], Local Fractional Continuous Wavelet Transform [15] and Local Fractional Discrete Wavelet Transform [16] are examples of more specific transforms applied on signals that are defined on cantor sets. Recurrence quantification analysis (RQA) [17] characterizes recurrence and returning behavior of a trajectory by using Recurrence Plot (RP) [18]. All of these features characterize properties of behavior of trajectories in phase space and each is used in many applications in physics, finance or engineering [19–26].

3. Method

This paper proposes a method to extract features from phase space of nonlinear systems. In this study, we focus on finding properties of trajectories that can present "volumetric behavior" of sequence of state vectors. Volumetric behavior characterizes occupied space and changes in occupied space of trajectory in space. First, we introduce the concept of phase space availability Section 2.1), and then we present a method to extract appropriate features (Section 2.2). This section is followed by describing these features (Section 2.3).

3.1. Trajectories in phase space

Dynamical systems are usually represented in three types: 1- phase space 2- time series 3- time-evolution law. In a d-dimensional phase space of a dynamic system at a fixed time t, the state of the system can be specified by d variables. These variables form vector $\vec{\mathbf{x}}(t)$:

$$\vec{\mathbf{x}}(t) = (x_1(t), x_2(t), \ldots, x_d(t))^T \tag{1}$$

For continuous-time systems, the evolution time is given by a set of differential equations. In fact, the evolution time law allows us to determine the state of the system at time t from the state at all previous times.

$$\vec{\mathbf{x}}\cdot(t) = \frac{d(\vec{\mathbf{x}}(t))}{dt} = F(\vec{\mathbf{x}}), \qquad F : R^d \rightsquigarrow R^d \tag{2}$$

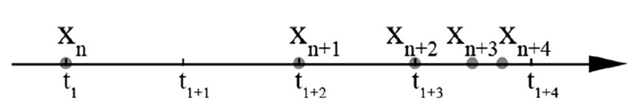
The vector $\vec{x}(t)$ defines a trajectory in d-dimensional phase space.

In an experimental setting, we do not often have access to all d states of phase space and a single discrete time measurement is available. In this case, phase space has to be reconstructed from time series $x(t) = \{x_1, x_2, ?, x_N\}$ [27]. Takens method [28] is frequently used for reconstructing phase space from time series $x(t)$ using two parameters embedding dimension μ and delay τ:

$$\vec{\mathbf{x}}_i(t) = (x_i, x_{i+\tau}, \ldots, x_{i+(\mu-1)\tau}), \qquad i = [1 \quad N-(\mu-1)\tau] \tag{3}$$

The false nearest-neighbors algorithm [29] and the mutual information [30] can be used for choosing appropriate dimension μ and delay τ parameters, respectively.

In next subsection, we propose a method to quantify volumetric behavior of trajectory.



$$T = \begin{pmatrix} 0 & 2 & 3 & 3.5 & 3.75 \\ 2 & 0 & 1 & 1.5 & 1.75 \\ 3 & 1 & 0 & 0.5 & 0.75 \\ 3.5 & 1.5 & 0.5 & 0 & 0.25 \\ 3.75 & 1.75 & 0.75 & 0.25 & 0 \end{pmatrix} \rightarrow T_1 = \begin{pmatrix} 2 & 3 & 3.5 & 3.75 \\ 2 & 1 & 1.5 & 1.75 \\ 3 & 1 & 0.5 & 0.75 \\ 3.5 & 1.5 & 0.5 & 0.25 \\ 3.75 & 1.75 & 0.75 & 0.25 \end{pmatrix}$$

$$\rightarrow T^* = \begin{pmatrix} -1 & -0.5 & -0.25 \\ 1 & -0.5 & -0.25 \\ 2 & 0.5 & -0.25 \\ 2 & 1 & 0.25 \\ 2 & 1 & 0.5 \end{pmatrix}$$

Fig. 2. An example of calculating matrix $\mathbf{T}^*$. x_n, x_{n+1}, x_{n+2}, x_{n+3} and x_{n+4} are stated in one-dimensional space. $\mathbf{T}$ is distance matrix that is provided by calculating distance of each pair of states. Matrix $\mathbf{T_1}$ is achieved by removing the main diagonal of matrix T. This matrix is converted to $\mathbf{T^*}$ by using Eq. (8).