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# Using 0–1 test to diagnose chaos on shape memory alloy dynamical systems

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ABSTRACT

Shape Memory Alloy (SMA) dynamical systems may exhibit a rich response that can include periodic, quasi-periodic, chaotic and hyperchaotic behaviors. In this regard, diagnostic tools are important in order to identify the different types of behaviors. This paper aims to analyze systems with SMA elements through a nonlinear dynamics perspective with a specific focus on the use of 0–1 test to quantify the chaoticity of the dynamical response of SMA oscillators. The investigation includes different constitutive models for the restitution force on both single- and two-degree of freedom oscillators. Results of the 0–1 test are compared with Lyapunov exponents calculated with different algorithms. The analyses show that the 0–1 test can be considered a reliable and computationally efficient alternative as a diagnostic tool of chaotic responses.



## 1. Introduction

Shape Memory Alloys (SMAs) are being used in several applications due to their remarkable thermomechanical behavior. Applied dynamics usually exploits SMA capacity to dissipate energy and to change properties due to solid phase transformations. Dynamical responses of SMA systems are very rich due to their strong nonlinearities. Because of that, the use of SMAs in dynamical applications requires a deep understanding of the system response [29].

Nonlinear dynamics investigations of SMA systems started in the 1990s [12,32] and since then, literature has several investigations treating their complex responses that include chaotic behavior. In general, numerical simulations are performed employing different constitutive models. The thermomechanical description of SMAs can be done in different ways and there exists several reviews of the state of the art about constitutive modeling, see e.g., Lagoudas [22] and Paiva et al. [27]. In this regard, the nonlinear dynamics analysis of SMA systems have some efforts that should be highlighted: Savi and Pacheco [31] employed the polynomial constitutive model treating both single and two-degree of freedom oscillators; Bernardini and Rega [4] employed the Bernardini–Pence's model; Savi et al. [30] employed the model with internal constraints [27]; Machado et al. [26] employed Boyd–Lagoudas' model. Besides, some experimental investigations attest the main conclusions related to numerical simulations: Enemark et al. [8,9]; Aguiar et al. [1]; Sitnikova et al. [33]; Machado [25].

The use of SMA to vibration reduction may be strongly influenced by the eventual presence of chaotic motions as they often occur in conjunction with strong jumps of response amplitude. Such jumps as well as the unpredictability of the response may drastically reduce, if not completely eliminate, the effectiveness of such devices. For this reason, reliable tools for chaos detection may be very important also in the design of SMA-based devices

Deterministic chaos is a possible response of SMA systems and a proper diagnose is one of the essential issues related to the system investigation. Lyapunov exponents constitute a well-established diagnostic tool for chaotic dynamical systems, and several algorithms can be employed to evaluate the Lyapunov spectrum. Lyapunov exponent calculation can be performed either from equations of motion or from time series. Concerning time series analysis, it is important to evaluate the robustness of each tech-

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nique due to noise contamination. In this regard, the algorithm due to Kantz [20] is a classical approach to estimate maximum Lyapunov exponent presenting low noise sensitivity. The algorithm due to Wolf et al. [36] is a classical approach for systems governed by ordinary differential equations that can be linearized around a reference trajectory. The hysteretic behavior of SMAs introduces difficulties for the application of this method. Machado et al. [26] proposed an approach to employ the algorithm due to Wolf et al. [36] on hysteretic systems.

The 0–1 test has been used as an interesting alternative to diagnose chaos in dynamical systems, being of special interest for systems where the classical approaches are difficult to be applied. In brief, this is a statistical approach based on the asymptotic properties of a Brownian motion chain. Gottwald and Melbourne [14,15] presented this procedure to distinguish chaotic from regular behavior in deterministic systems. The test provides, as a result, a number that lies between 0 and 1. If the dynamical behavior of the tested system is chaotic, the result is close to 1, or close to 0 if the system exhibits non-chaotic or regular behavior. Gottwald and Melbourne [18] and Bernardini and Litak [2] presented general overviews of the theoretical background and the use of 0–1 test for chaos diagnose.

The 0–1 test can be applied directly to time series and therefore is independent on the nature of the underlying dynamical system [16,17]. Litak et al. [23] and Bernardini et al. [7] applied the 0–1 test to SMA systems considering time series obtained from numerical simulations of equations of motion. Different applications of the test can be found in several research efforts. Falconer et al. [10] applied the test to an experimental time series from a bipolar motor. Webel [35] employed this method for testing chaos in the return time series from the German stock market. Other interesting applications of the 0–1 test can be found on Krese and Govekar [21] and Yuan et al. [38].

This paper discusses the application of the 0–1 test to SMA systems and investigates its effectiveness in the detection of chaotic responses establishing a comparison with the Lyapunov exponents. Moreover, since SMA exhibits a complex thermomechanical response and several constitutive models have been proposed in the literature, the analysis includes the performances of the diagnostic tools on three different SMA models: polynomial model [11,31]; model with internal constraints [27,30], Bernardini–Pence's model [3–6]. Moreover, single- and two-degree of freedom systems are analyzed allowing one to obtain a proper comprehension of the general behavior of SMA systems, investigating different system dimensions. It is beyond the scope of this contribution the comparison of the SMA models. Time series are generated from the equations of motion and results obtained with the 0–1 test are compared with Lyapunov exponents. Basically, the algorithms due to Wolf et al. [36] and Kantz [20] are employed for the estimation of the exponents. The main goal is to evaluate the 0–1 test capacity to diagnose different kinds of response.

Two different archetypal systems are evaluated considering distinct dimensions: single-degree of freedom system, 1-dof (Fig. 1a); two-degree of freedom system, 2-dof (Fig. 1b). Essentially, the single-degree of freedom system is an oscillator with a mass, $m$, with a displacement $u$, connected to the support by an SMA element and a linear viscous damper with coefficient $c$, and subjected to a harmonic excitation $F = \bar{F}\sin(\Omega t)$. The two-degree of freedom system consists of two coupled oscillators with masses, $m_i$ ($i = 1,2$), connected by SMA elements and linear dampers with coefficient $c_i$ ($i = 1,2,3$). Each mass has displacement $u_i$ ($i = 1,2$) being harmonically excited by an external force $F_i = \bar{F}_i\sin(\Omega_i t)$ ($i = 1,2$).

After this introduction, the paper is organized as follows. Initially, a brief description of the diagnostic tools is presented, emphasizing Lyapunov exponents and 0–1 test. Numerical simulations are then carried out for different models, starting with single-degree of freedom systems. Polynomial, internal constraints and Bernardini–Pence models are treated. Afterwards, a two-degree of freedom system described with polynomial model is investigated. Concluding remarks are then discussed.

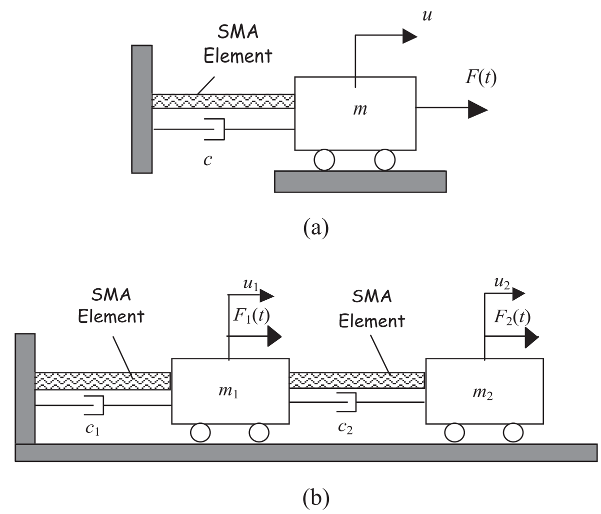


**Fig. 1.** SMA dynamical systems. (a) single and (b) two degree-of-freedom mass systems.

## 2. Diagnostic tools

Nonlinear dynamics of SMA systems is very rich, being associated with complex responses. In this regard, periodic, quasiperiodic, chaotic and hyperchaotic solutions can arise and it is important to employ suitable diagnostic tools that allow a proper identification of these behaviors. Usually, the estimation of some system invariant is adopted and the most widely used is the Lyapunov exponents.

The classical algorithm due to Wolf et al. [36] can be considered a well-established procedure when equations of motion are available. Nevertheless, its use needs the determination of a system linearization that is not an easy task for hysteretic systems, where complex, generally non-smooth, constitutive equations are usually employed. In this regard, there is the alternative of a time series analysis where Kantz [20] algorithm is an interesting approach. The 0–1 test proposed by Gottwald and Melbourne [14] is another interesting alternative for time series analysis.

### 2.1. Lyapunov exponents

The set of Lyapunov exponents is a system invariant that estimates its sensitivity to initial conditions by evaluating local divergence of nearby orbits. It represents one of the most accepted diagnostic tool for chaos. In brief, the divergence of nearby orbits can be analyzed monitoring the distance between a reference orbit and its neighboring orbits while the system evolves through time. If the measured distance increases, there is a local divergence that characterizes chaos. Chaotic response is, therefore, associated with at least one positive value, representing a divergent direction.

Usually, the reference orbit is evaluated from the equations of motion and the nearby orbit evolution is monitored by an extension of the equations of motion. Wolf et al. [36] presented a procedure where this extension is evaluated from a linearized version of the dynamical system. Besides, new initial conditions are adopted for each time step, avoiding an explosive behavior.

The algorithm due to Kantz [20] employs a similar idea where the distances between two orbits increase with a rate given by the