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Nonlinear dynamics of equity, currency and commodity markets in the aftermath of the global financial crisis

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ABSTRACT

We attempt to quantify the intrinsic nonlinear dynamics of thirty international financial markets. Fractality, chaoticity and randomness are explored during and after the recent global financial crisis. We find that most markets exhibited persistent long-range correlations during the crisis, whilst anti-persistent patterns are identified after the crisis. Moreover, the nonlinear dynamics in all markets do not exhibit chaotic features. Importantly, the degree of randomness has increased in most of markets in the aftermath of the crisis. Overall, the nonlinear characteristics of the temporal dynamics of the major financial markets have been notably modified in the post-crisis period.

1. Introduction

Measuring complexity in financial assets is of paramount importance in order to comprehend their inherent nonlinear dynamics. Various techniques from statistical mechanics and physics have been applied to financial time series in an attempt to capture fractality, chaos, randomness or mixed dynamics. Without a doubt, there is no convincing argument towards the assumption that asset prices or returns exhibit linear dynamics. Measuring long-range memory has been investigated via a plethora of studies and methods in the literature i.e., through detrended fluctuation analysis (DFA) [1], multifractal detrended fluctuation analysis (MF-DFA) [2], generalized Hurst exponent (GHE) [3], weighted generalized Hurst exponent (wGHE) [4] or a combination of the above [5–18]. Obviously, examining the temporal correlation in financial data could reveal the nature of the dynamical fluctuations.

Chaos theory [19] is also appealing in order to understand the micro-behavior of agents in financial markets. A chaotic dynamical system is sensitive to initial conditions, exhibits non-periodic movements, and consequently is not predictable in the long run. In this regard, several works have been conducted to scrutinize the existence of chaos in financial markets. For instance, stock markets [7,20,21], crude oil [22] and currency markets [23–27] have been investigated mostly utilizing Hurts and Lyapunov exponents [28,29]. In the same vein, measuring entropy could be valuable towards detecting randomness in the underlying processes of financial series. Entropic statistics allow revealing the (a)proximity of the trajectories of a system and their inherent randomness. Because of their informational content, entropy-based measures were recently employed to assess the stability of equity [29–32], FX [33,34] and gold markets [34] accordingly.

The main purpose of our study is to detect the nature, dimensionality and direction of complexity in various financial markets based on the characterization of their nonlinear dynamics during and after the global financial crisis. Specifically, the intrinsic patterns of the time series employed are quantified via the use of the Hurst exponent (HE), the largest Lyapunov exponent (LLE) and the Renyi entropy measure. Basically, we attempt to estimate the long-range correlation, divergence and randomness of the systems that optimally explain the behavior of the investigated markets. Fractality, chaoticity and stochasticity are all explored to assess the nonlinear features associated with each underlying series and its oscillations. In our work, HE is estimated by means of the detrended fluctuation analysis (DFA) [1], LLE is estimated using artificial neural networks [26] while the Renyi entropy [35] is computed to quantify any existing randomness.

Overall, we contribute to the relevant literature in the following ways: we describe the inherent characteristics of complexity in numerous major financial markets by using for the first time three different statistical concepts derived from statistical mechanics. Secondly, we assess how temporal dynamics evolved during and after the crisis period; clearly there is a growing interest on this topic [5,8,22,36,37]. More importantly, we conduct an exhaustive analysis involving a very large number of stock, cur-

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rency and commodity markets, thus significantly enrich the existing econophysics literature [5–37]. The rest of our paper is sketched as follows: Section 2 describes the implemented methods, while Section 3 presents the data and discusses the empirical results. Finally, Section 4 concludes.

2. Methodology

2.1. Detrended fluctuation analysis (DFA)

The DFA [1] is capable of detecting long-range dependence in nonstationary data. The methodology applied to a signal (series) y is described as follows:

a) Define the suite x_N of the cumulative series of the original y_i fluctuations about its mean:

$$x_N = \sum_{i=1}^{N} (y_i - \bar{y}) \tag{1}$$

b) Divide x_N into boxes of equal length n.
c) In each box, fit the local trend of x_N by a polynomial $P(n,N)$ that represents the local trend of the box. In the present study, a polynomial of degree one is employed.
d) For the given n box size, compute the root-mean-squared detrended fluctuation of the signal x_N:

$$F(n,N) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - P(n,N))^2} \tag{2}$$

For each of the available n box size, the last step is repeated to obtain the empirical relationship between the overall fluctuation $F(n,N)$ and the box size n:

$$F(n,N) \propto n^H \tag{3}$$

The Hurst exponent H is estimated by running a regression of the $\log(F(n,N))$ upon $\log(n)$. For $H = 0.5$, the dynamics of the original time series follow a random walk. For $0 < H < 0.5$, the series are anti-persistent, while on the contrary the series are persistent in case $0.5 < H < 1$.

2.2. Largest Lyapunov exponent

The Lyapunov exponent is basically an indicator of the qualitative behavior in a dynamical system based on a one-dimensional time series analysis. In particular, the Lyapunov exponent allows for determining whether a given dynamical system has divergent or convergent trajectories. Assume a noisy chaotic system of time series x_t given by:

$$x_t = f(x_{t-L}, x_{t-2L}, \ldots, x_{t-mL}) + \varepsilon_t \tag{4}$$

where L is the time delay, m is the embedding dimension, ε a noise term, f an unknown function used to approximate a chaotic map, and t the time script. A noise-free system has zero variance, i.e., $Var(\varepsilon_t) = 0$. Then, the Lyapunov exponent λ of a noisy chaotic system is estimated as follows [26,38]:

$$\lambda = \lim_{M\to\infty} \frac{1}{2M} \log(\upsilon_1) \tag{5}$$

where υ_1 is the largest eigenvalue of the matrix $T'_M T_M$ where T_M is given in [26,38]:

$$T_M = \prod_{t=1}^{M-1} J_{M-1} \tag{6}$$

We denote $M \leq T$ as the block-length of equally spaced evaluation points, and J as the Jacobian matrix of the chaotic map f. The Jacobian J at a starting point x_0 is expressed as follows:

$$J^t(x_0) = \frac{df^t(x)}{dx}\Big|_{x_0} \tag{7}$$

A multilayer feed-forward neural network trained via a gradient descent algorithm [26,38] can be employed to approximate the unknown chaotic map f:

$$x_t \approx \alpha_0 + \sum_{j=1}^{q} \alpha_j A\left(\beta_{0,j} + \sum_{i=1}^{m} \beta_{i,j} x_{t-iL}\right) + \varepsilon_t \tag{8}$$

where q is the number of hidden layers, α_j are the layer connection weights, α_0 the network bias and A the sigmoid function as in [26,38]. The triplet (L, m, q) is set at high values and varies until the largest Lyapunov exponent (LLE) is obtained following the method presented in [38]. Evidently, when $\lambda \geq 0$ the underlying time series possess incorporates chaotic dynamics, while instead when $\lambda < 0$ we have an indication of convergence between close trajectories. In the latter case classic attractors exist.

2.3. Renyi entropy

Renyi entropy [35] is employed to measure the degree of randomness in each market. The most prominent feature of this specific entropic estimator is its robustness to heavy-tailed distributions that occur in real-world complex dynamical systems such as those described by financial time series [39]. For a time series $\{x_t\}_{t=1}^{n}$ the Renyi entropy is given as:

$$R_q(x) = \frac{1}{1-q}\log\left(\sum_{i=1}^{n} p_i^q\right) \tag{9}$$

where $q \geq 1$ and $q \neq 1$ and p_i is the discrete probability such that $\sum_i p_i = 1$. The parameter q is a diversity index of different probabilities. In particular, a high order-q allows focusing on extreme events with low probability. Otherwise, a low order-q describes regular events with higher probability. As we are not interested in measuring entropy at different scales by varying the q and not focusing on singular events, the parameter q is set to 2 [35].

3. Data and empirical results

We consider the daily prices for thirty international equity, currency and commodity markets for a period spanning 15 January 2007 to 19 December 2016. In particular, we examine the stock markets of USA, UK, France, Germany, Japan, Canada, China, Spain, Greece, Belgium, Switzerland, Portugal, Italy and Taiwan, the currency markets for US/Euro, US/UK, US/CHF, and the commodity series of the Crude oil, Harbor, Natural gas, Copper, Platinum, Silver, Gold, Palladium, Corn, Coffee, Cocoa, Cotton and Wheat. The data is obtained from Datastream International database. The investigated sample period is classified in two disjoint groups: the crisis period i.e., 15 January 2007 to 31 December 2010, and the post-crisis sample namely 1 January 2011 to 19 December 2016. We conduct our analysis based on the first-difference of the logarithm of the prices for all markets.

Table 1 depicts the estimated Hurst (HE) and largest Lyapunov exponents (LLE) as well as the Renyi entropy (RE) measure for all markets and periods involved. We observe that all HEs are higher than 0.5 during the crisis, except those of silver, gold, coffee and cocoa. Instead, most of the calculated HEs obtain a value below 0.5 after the crisis except for Hang Seng, ATHEX, crude oil, gold and cotton markets. Furthermore, we find that the estimated HEs of PSI, TAIEX, harbor, silver and corn are slightly above 0.5. Subsequently, while the majority of the markets demonstrate a persistent long-range behavior during the crisis, yet they reveal anti-persistent long range correlations after the crisis period. In addition, Table 1 shows that all estimated LLE are negative in any period, thereby none of the markets exhibits chaotic dynamics. Instead, each one presents convergent trajectories. Finally, all markets display low-entropy statistics during the crisis in comparison to the ones after the crisis, perhaps with the exceptions of