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Bifurcation and chaos in time delayed fractional order chaotic memfractor oscillator and its sliding mode synchronization with uncertainties

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ABSTRACT

A novel chaotic memfractor oscillator with one unstable equilibriums is proposed. Various dynamic properties of the proposed system are derived and investigated to show the existence of chaotic oscillations. The fractional order time delayed model of the chaotic memfractor oscillator is derived considering time delay in the memcapacitor. Bifurcation of the time delayed system with its delay factor is investigated along with the parameter space bifurcation. A novel methodology for synchronizing identical time delayed systems with an uncertainty in the slave system is proposed and tested with the proposed time delayed fractional order chaotic memfractor oscillator.

1. Introduction

The fourth circuit element popularly known as memristors was first postulated by Chua in 1971 [1]. Until 2008 when researchers of HP labs fabricated a solid state implementation of memristor, none was known much about memristor realization [2]. Since then many other memristor models have been introduced [3–6]. Memristors are considered to be highly nonlinear with non-volatile characteristics and can be implemented with nano-scale technologies [3–6]. To design memristor oscillators, a new kind of nonlinear circuits with oscillatory memories and periodically forced flux controlled memductance models are investigated [7,8].

Memristor based chaotic oscillators are widely investigated in the last one decade. Circuits with two HP memristors in antiparallel is demonstrated showing a variety of chaotic attractors for different values of components [9]. A current feedback op amp based memristor oscillators is analyzed and simulation results are investigated [10]. A simple autonomous memristor based oscillator with external sinusoidal excitation is used to generate chaotic oscillations. A discrete model for this HP memristor is derived and implemented using DSP chips [11] implementing memristor. Recently a new hyper chaotic system with two memristors is investigated and its application to image encryption is analyzed. The correlation and ant attack capability between adjacent pixels are investigated [12].

Practical implementation of memristor based chaotic circuits with off the shelf components is desired for real time applications [13]. Memristor based chaotic circuit for pseudo random number generation are analyzed with applications to cryptography [14]. Memristor based chaotic circuits for text and image cryptography is investigated and the correlation analysis shows the effectiveness of the proposed cryptographic scheme over other encryption algorithms [15]. Memcapacitor based chaotic circuits with a HP memristor is proposed and analysis the proposed oscillator is implemented in DSP for further applications [16].

Recently many researchers have discussed about fractional order calculus and its applications [19–21]. Fractional order nonlinear systems with different control approaches are investigated [22–24]. Fractional order memristor based no equilibrium chaotic and hyperchaotic systems are proposed [17,18,40,41]. A novel fractional order no equilibrium chaotic system is investigated in [25] and a fractional order hyper chaotic system without equilibrium points is investigated in [26]. Memristor based fractional order system with a capacitor and an inductor is discussed [27]. Numerical analysis and methods for simulating fractional order nonlinear system is proposed in [28] and matlab solutions for fractional order chaotic systems is discussed in [29].

Many synchronization methodologies have been reported in literatures [68–70]. The synchronization of chaotic and hyperchaotic systems has many applications like secure communications [31–32], cryptosystems [33–34], etc. Synchronization of fractional order chaotic systems for orders between $1 \le q \le 2$ was discussed in [30]. Adaptive synchronization of the chaotic and hyperchaotic

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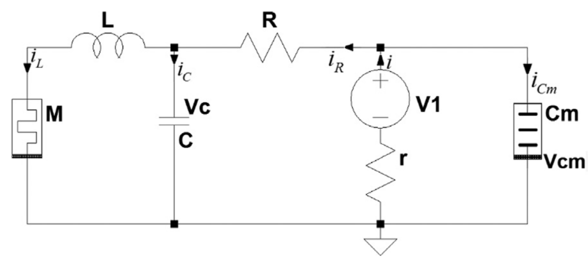

Fig. 1. Memcapacitor based chaotic oscillator.

systems with unknown parameters using sliding mode control and PID control are also discussed optimized by genetic algorithm [17,18,35–37]. Finite-time chaos synchronization of fractional-order chaotic and hyperchaotic systems using fractional nonsingular terminal sliding mode technique was also discussed in literature [41]. Most of the error dynamics stability is analyzed with Lyapunov stability theory [38,39]. Various methods of fractional order time delayed synchronization with sliding mode [63], active control [64], ring connection synchronization [65], lag synchronization [66] and generalized synchronization [67] was also discussed in the literatures.

The three main approaches derived to solve fractional-order chaotic systems are frequency-domain method [42], Adomian Decomposition Method (ADM) [43,47,48] and Adams–Bashforth–Moulton (ABM) algorithm [44]. The frequency-domain method is not always reliable in detecting chaos behavior in nonlinear systems [45]. On the other hand, ABM and ADM are more accurate and convenient to analyze dynamical behaviors of a nonlinear system. Compared with the ABM, ADM yields more accurate results and needs less computing resources as well as memory resources [46].

2. Problem formulation

Several memcapacitor models, including piecewise linear, quadric and cubic function models, memristor-based memcapacitor models are discussed in several literatures [51–54]. Some special phenomena such as hidden attractors, coexistence attractors, and extreme multistability were found in memcapacitor based chaotic oscillators [55–57].

In this paper we investigate a novel chaotic memfractor oscillator (CMO) with charge controlled memcapacitor and flux controlled memristor as shown in Fig. 1.

R is the resistance,$L$is the inductances,G is the conductance r is the internal resistance of the voltage source and C is the capacitance. C_m is the memcapacitor [49,50] and M is the flux controlled memristor [9–16] . The current flowing through the circuit arei_G, i_R, i_{C_m}, i_L. The relationship between the voltage $v_{C_m}(t)$and the charge $q_{C_m}(t)$of the memcapacitor is defined as,

$$v_{Cm}(t) = (\alpha + \beta\sigma_{Cm})q_{Cm}(t) \tag{1}$$

where, $\frac{d\sigma}{dt} = q_{C_m}(t)$.Applying Kirchhoff's Law to the circuit shown in Fig. 1, we derive the five state equations of the circuit as,

$$\begin{aligned}
\frac{d\sigma}{dt} &= q_{C_m}(t) \\
\frac{dq_M}{dt} &= i_L \\
\frac{dq_{Cm}}{dt} &= \frac{v_{C_m}}{r} + \frac{1}{R}(v_c - v_{C_m}) \\
\frac{di_L}{dt} &= \frac{1}{L}(V_C - Mi_L)
\end{aligned}$$

$$\frac{dv_c}{dt} = \frac{1}{c}\left(-i_L + \frac{1}{R}(v_{C_m} - v_c)\right) \tag{2}$$

Using $x=\sigma, y=q_M, z=q_{C_m}, u=i_L, v=v_cande=\frac{1}{L}, f=\frac{1}{C}, g=\frac{1}{R}, h=\frac{1}{r}$, and with the memristor flux elements as$a=0.01, b=0.01$, memcapacitor $c=0.7, d=-0.8$ and $L=0.136H, C=58.82F, R=0.2\Omega, G=2.1$, we arrive at the fifth order dimensionless mathematical model of the memfractor oscillator system as

$$\begin{aligned}
\dot{x} &= z \\
\dot{y} &= u \\
\dot{z} &= a_1 z + a_2 xz + a_3 v \\
\dot{u} &= a_4 v + a_5 u(1-y) \\
\dot{v} &= a_6 u + a_7 xz + a_8 z + a_9 v
\end{aligned} \tag{3}$$

with $a_1=-1.89$; $a_2=-2.16$; $a_3=4.8$; $a_4=7.35$; $a_5=-0.0735$; $a_6=-0.17$; $a_7=0.6528$; $a_8=0.571$; $a_9=-0.816$. Fig. 2 shows the 2D phase portraits of the CMO system for the initial conditions [0, 0, 0, 0, 0.01].

3. Dynamic analysis of chaotic memfractor oscillator (CMO)

The dynamic properties of the CMO system such as dissipativity, equilibrium points, eigen values, Lyapunov exponents and Kaplan–Yorke dimension are derived and discussed in this section.

3.1. Equilibrium points

By equating $\dot{X}=0$, the CMO system (3) shows only one equilibrium point at origin (E_1). The Jacobian matrix of the CMO system (3) is

$$J(X) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ a_2 z & 0 & a_1 + a_2 x & 0 & a_3 \\ 0 & -a_5 u & 0 & a_5 - a_5 y & a_4 \\ a_7 z & 0 & a_8 + a_7 x & a_6 & a_9 \end{bmatrix} \tag{4}$$

The characteristic equation of the system is derived as,

$$\lambda^5 + (-a_1 - a_5 - a_9)\lambda^4 + \begin{pmatrix} a_1 a_5 + a_1 a_9 - a_4 a_6 \\ -a_3 a_8 + a_5 a_9 \end{pmatrix}\lambda^3 + \begin{pmatrix} a_1 a_4 a_6 - a_1 a_5 a_9 \\ +a_3 a_5 a_8 \end{pmatrix}\lambda^2 \tag{5}$$

and at equilibrium E_1 the characteristic equation is

$$\lambda^5 + 2.7795\lambda^4 + 0.248871\lambda^3 + 2.27339028\lambda^2 \tag{6}$$

and the corresponding Eigen values are

$$\lambda_1 = -2.9555,\ \lambda_{2,3} = 0.0880 \pm 0.8726i,\ \lambda_{4,5} = 0 \tag{7}$$

and $\lambda_{2,\,3}$ is the saddle focus. As per Routh–Hurwitz criterion, all the principal minors need to be positive for the CMO system to be stable. The principal minors are,

$$\Delta_1 = \delta_1 > 0,\ \Delta_2 = \begin{vmatrix} \delta_1 & \delta_0 \\ \delta_3 & \delta_2 \end{vmatrix} > 0,\ \Delta_3 => \begin{vmatrix} \delta_1 & \delta_0 & 0 \\ \delta_3 & \delta_2 & \delta_1 \\ 0 & 0 & \delta_3 \end{vmatrix} > 0 \tag{8}$$

$$\Delta_4 = \begin{vmatrix} \delta_1 & \delta_0 & 0 & 0 \\ \delta_3 & \delta_2 & \delta_1 & \delta_0 \\ 0 & \delta_4 & \delta_3 & \delta_2 \\ 0 & 0 & 0 & \delta_4 \end{vmatrix} > 0,\ \Delta_5 = \begin{vmatrix} \delta_1 & \delta_0 & 0 & 0 & 0 \\ \delta_3 & \delta_2 & \delta_1 & \delta_0 & 0 \\ 0 & \delta_4 & \delta_3 & \delta_2 & \delta_1 \\ 0 & 0 & \delta_5 & \delta_4 & \delta_3 \\ 0 & 0 & 0 & 0 & \delta_5 \end{vmatrix} > 0 \tag{9}$$

where $\delta_0 = 1, \delta_1 = -a_1 - a_5 - a_9, \delta_2 = a_1 a_5 + a_1 a_9 - a_4 a_6 - a_3 a_8 + a_5 a_9, \delta_3 = a_1 a_4 a_6 - a_1 a_5 a_9 + a_3 a_5 a_8 \delta_4 = 0, \delta_5 = 0.$