Chaos, Solitons and Fractals 103 (2017) 357–363

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Synchronization of fractional-order complex dynamical networks via periodically intermittent pinning control

Hong-Li Li[a,*], Cheng Hu[a], Haijun Jiang[a], Zhidong Teng[a], Yao-Lin Jiang[a,b]

[a] College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China
[b] Department of Mathematics, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 27 March 2017
Revised 3 June 2017
Accepted 19 June 2017

Keywords:
Fractional-order
Complex networks
Synchronization
Pinning control
Intermittent control

ABSTRACT

This paper investigates the synchronization problem of general complex networks with fractional-order dynamical nodes. Pinning state feedback controllers have been proved to be effective for synchronization control of fractional-order complex networks. We will show that pinning intermittent controllers are also effective for synchronization control of general fractional-order complex networks. Based on the Lyapunov method and periodically intermittent control method, several low-dimensional criteria are derived for the synchronization of such dynamical networks. Finally, a numerical example is presented to demonstrate the validity and feasibility of the theoretical results.

1. Introduction

Complex networks widely exist everywhere and have been studied by researchers in various fields, such as the Internet networks [1], epidemic spreading networks [2], biological networks [3], neural networks [4], social networks [5], etc. A network consists of a large number of nodes and the connections between them. These nodes have different meanings in different situations, such as computers, patches, etc. In many practical applications, all the nodes of a network are desired to synchronize with a given orbit. However, due to the complexities of node dynamics and topological structure of a network, all the nodes can not achieve the goal by themselves in many situations [6]. Then, some proper controllers should be adopted to realize the desired goal. Up to now, some control schemes have been introduced to realize the network synchronization, such as adaptive control [7], variable structure control [8], pinning control [9], impulsive control [10], intermittent control [11], and so on.

Intermittent control, which was first introduced to control linear econometric models in [12], has been widely used in engineering fields, such as manufacturing, transportation and communication, and so on [13–15] Usually, the control time is periodic, and in any period, the time is composed of work time (or control time) and rest time, which can be found in the map (see Fig. 1) [16]. Compared with impulsive control, periodically intermittent control is easier to be implemented due to it has a nonzero control time. The advantage of pinning control is that this strategy can control a given network by pinning a small fraction of all nodes of the network [17]. From the perspective of economic costs, pinning control can reduce the number of controllers in complex networks and periodically intermittent control can shorten work time. Therefore, if we combine two kinds of control methods together, the control cost can greatly be saved [18–20]. In [19], Cai et al. investigated exponential synchronization of complex networks via pinning periodically intermittent control. In [20], Liu et al. considered the cluster synchronization of complex networks via periodically intermittent pinning control.

It should be noted that most of the existing results about periodically intermittent pinning control are mainly concentrated on the integer-order systems, see [18–20] and references therein. As we know, fractional-order models provide an excellent instrument for the description of memory and hereditary properties of various materials and dynamical processes. In fact, real-world processes generally or most likely are fractional-order systems, for example, viscoelastic systems, dielectric polarization, electromagnetic waves, heat conduction, robotics, biological systems, quantitative finance and so on [21–23] It would be far better if many practical problems are described by fractional-order dynamical systems rather than integer-order ones. Not surprisingly, many researchers have already introduced fractional calculus to complex networks, and some interesting results about stability and synchronization

* Corresponding author.
E-mail addresses: lihongli3800087@163.com (H.-L. Li), wacheng2003@163.com (C. Hu), jianghaijunxju@163.com (H. Jiang), zhidong_teng@sina.com (Z. Teng), yljiang@mail.xjtu.edu.cn (Y.-L. Jiang).

http://dx.doi.org/10.1016/j.chaos.2017.06.025
0960-0779/

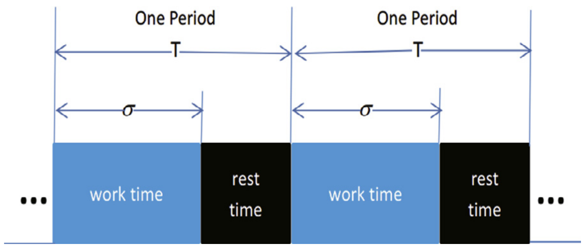

Fig. 1. Sketch map of periodically intermittent control.

of fractional-order complex networks have been obtained [24–31]. In [30], Wang et al. investigated the synchronization of fractional-order complex networks via pinning control. In [31], Wang et al. studied the synchronization of fractional-order complex networks via pinning impulsive control. To the best of our knowledge, there is seldom work so far dedicated to the investigation of intermittent pinning control for fractional-order complex networks. In this paper, we investigate synchronization of fractional-order complex networks via periodically intermittent pinning control.

The main contributions of this paper are the following three aspects: (1) Intermittent pinning control, which is first introduced to control the fractional-order complex network. (2) Some low-dimensional criteria are derived to achieve synchronization of the considered network. (3) We give some steps to design controllers for the considered network.

This paper is organized as follows. In Section 2, the model of complex network is formulated and some preliminaries are presented. In Section 3, some novel synchronization criteria for fractional-order complex network are obtained via periodically intermittent pinning control. In Section 4, a numerical example is provided to illustrate the effectiveness of the theoretical results. Finally, conclusions and future research topics are given in Section 5.

2. Model description and preliminaries

In this paper, let $\mathbb{R}^n$ be the n-dimensional Euclidean space. For vector $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, $\|x\| = (x_1^2 + x_2^2 + \dots + x_n^2)^{1/2}$ denotes the Euclidean norm. Throughout the paper, we use the Caputo fractional-order derivative because the initial conditions of fractional differential equations with Caputo derivatives take on the same form as for integer-order ones, which have more applications in modeling and analysis. In this section, some definitions and lemmas are recalled which will be needed later.

Definition 1 [23]. The fractional integral of order q for a function f is defined as

$$ {}_{t_0}I_t^q f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} f(s) ds, \quad q > 0, $$

where $\Gamma(\cdot)$ is the well-known Gamma function.

Definition 2 [23]. Caputo fractional derivative of order q for a function $f \in C^n([t_0, +\infty), \mathbb{R})$ is defined by

$$ {}_{t_0}^{c}D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{q-n+1}} ds, $$

where $\Gamma(\cdot)$ is the Gamma function, $t \geq t_0$ and n is a positive integer such that $n-1 < q < n$. Particularly, when $0 < q < 1$,

$$ {}_{t_0}^{c}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^{q}} ds. $$

Consider a general complex network consisting of N identical dynamical nodes with linearly diffusive couplings, which can be described as follows:

$$ {}_0^cD_t^q x_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \quad i = 1, 2, \dots, N, \tag{1} $$

where $0 < q < 1$, ${}_0^cD_t^q$ is in the sense of the Caputo fractional derivative, $x_i(t) = (x_i^1(t), x_i^2(t), \dots, x_i^n(t))^T \in \mathbb{R}^n$ is the state vector of the ith node, and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear vector function, $c > 0$ is the coupling strength. $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \dots, \gamma_n) > 0$ is the inner connecting matrix. $A = (a_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network, in which a_{ij} is defined as follows: If there is a direct connection from node i to node j, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$, and the diagonal elements of matrix A are defined by $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$.

In addition, we have the following assumption.

Assumption 1. The graph corresponding to the network (1) is strongly connected and balanced.

Remark 1. The coupling configuration matrix A represents the topological structure of network (1). The graph corresponding to the network (1) is strongly connected if and only if its coupling configuration matrix A is irreducible [32]. The graph corresponding to the network (1) is said to be balanced if $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$ for all $i \in \{1, 2, \dots, N\}$, see [33].

Let $s(t) \in \mathbb{R}^n$ be a solution of an isolated node system

$$ {}_0^cD_t^q s(t) = f(s(t)). \tag{2} $$

The main objective of this paper is to apply pinning periodically intermittent control scheme to make the states of network (1) globally synchronize with $s(t)$, in the sense that

$$ \lim_{t\to+\infty} \|x_i(t) - s(t)\| = 0, \quad i = 1, 2, \dots, N, $$

for any initial conditions.

For this purpose, some intermittent controllers are added to partial nodes of network (1). Without loss of generality, suppose the first $l (1 \leq l < N)$ nodes are selected to be pinned, then we have the following controlled dynamical network:

$$ \begin{cases} {}_0^cD_t^q x_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i(t), & 1 \leq i \leq l, \\ {}_0^cD_t^q x_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), & l+1 \leq i \leq N, \end{cases} \tag{3} $$

where $u_i(t)$ is an intermittent periodical controller, which is described by

$$ u_i(t) = -d_i(t)(x_i(t) - s(t)) \tag{4} $$

with

$$ d_i(t) = \begin{cases} -d_i, & t \in [mT, mT+\sigma], \quad 1 \leq i \leq l, \\ 0, & t \in (mT+\sigma, (m+1)T), \quad 1 \leq i \leq l, \end{cases} $$

here $d_i > 0$ is a positive constant called control gain, $T > 0$ is the control period, $\sigma > 0$ is called the control width (control duration), and $m = 0, 1, 2, \dots$.

Let $e_i(t) = x_i(t) - s(t)$ $(1 \leq i \leq N)$ be synchronization errors, and $\delta = \sigma/T$ be the ratio of the control width σ to the control period T called control rate. According to (2), (3) and the control