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Optimal dynamic asset-liability management with stochastic interest rates and inflation risks

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ABSTRACT

This paper considers an optimal asset-liability management problem with stochastic interest rates and inflation risks under the expected utility maximization framework, where the stochastic interest rate follows the Hull-White interest rate model and the inflation risk is modelled by an additional stochastic process. The investor can invest in $n+1$ assets: cash, a default-free zero-coupon bond, an inflation-indexed bond and $n-2$ stocks. The liability process is given by a geometric Brownian motion rather than a Brownian motion to ensure a definite liability value. Applying the stochastic control theory and partial differential equation approach, we obtain the explicit solutions of optimal investment strategies for the power utility and exponential utility functions. We also provide numerical examples to show the effects of model parameters on the optimal investment strategies.

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1. Introduction

Establishing an effective asset-liability management (ALM) model is essential not only for institutional investors such as banks, pension funds and insurance companies, but also for individual investors who have the ability to increase their wealth through borrowing such as banks and credit institutions. Thus, ALM problems have attracted more and more attentions in the actuarial and academic literatures. Sharpe and Tint [1] first apply the portfolio selection techniques to consider an ALM problem in a static mean-variance (M-V) framework. Along with the breakthrough and progress in solving dynamic M-V formulations, many scholars adopt the M-V model to study the ALM problems. See for example, Leippold et al. [2], Chiu and Li [3], Chen et al. [4], Yan [5], Chiu and Wong [6] and so on. Moreover, the safety-first ALM is closely related to the M-V ALM. Recently, there are some researches about the safety-first ALM problems such as Chiu and Li [7] and Chiu and Wong [8].

It is well known that the mean-variance objective is not flexible enough to cope with the effect of an investor's risk aversion. Furthermore, the utility theory, and especially the expected utility theory introduced by von Neumann and Morgenstern [9] has demonstrated a far-reaching impact in finance and has been recognized for its usefulness and applicability in financial modelling. In recent years, optimal ALM problems based on the utility maximization criteria have attracted some attentions. Under the expected utility maximization framework, Rudolf and Ziemba [10] and Hoevenaars et al. [11] investigate a continuous-time ALM problem and a discrete-time ALM problem, respectively. In their models, the optimal investment strategies for the constant relative risk aversion (CRRA) utility function are obtained. Chen [12] studies a continuous-time ALM problem in the regime switching setting and derives the corresponding optimal investment strategies. Chiu and Wong [13] consider the ALM problem with stochastic interest rate under the CRRA utility framework, where the liability follows a risk model of compound Poisson process and the interest rate takes the Cox–Ingersoll–Ross (CIR) model.

Since the ALM plan for an investor may involve quite a long period, it is reasonable to take the risks of interest rate and inflation into account, especially inflation risk. Recently, there are some researches about the portfolio selection problem with stochastic interest rates and inflation risks. Battocchio and Menoncin [14] consider the optimal pension management in a stochastic framework by using the stochastic dynamic programming approach. Ma [15] corrects the defects of Battocchio and Menoncin [14]. But, there is still a problem in his model. With the setting of Battocchio and Menoncin [14] and Ma [15], there are equity risk, interest rate risk and inflation risk in the financial market while only two assets (stock and zero-coupon bond) can be independently determined,

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which implies that the investor cannot hedge against the inflation risk according to his risk-aversion. In the financial market, there are also some financial instruments used to hedge against the inflation risk such as inflation-indexed bonds (i.e., Treasury Inflation Protected Securities (TIPS) and UK inflation-linked gilt-edged securities (ILGS)). Fischer [16] studies the demand for inflation-indexed bonds by solving the inter-temporal optimization problem under the inflation risk and highlights the role of inflation-indexed bonds. Campbell and Viceira [17], Brennan and Xia [18], Chiarella et al. [19], and Kwak and Lim [20] probe the importance of inflation-indexed bonds for the long-term and conservative investment. By introducing inflation-indexed bonds, Han and Hung [21] focus on the optimal investment problem for the defined contribution (DC) pension plan with the risks of interest rate and inflation and draw a conclusion that the inflation-indexed bond is indispensable for the pension plan to hedge against the inflation risk and to provide a downside protection with the annuitants. Later, Guan and Liang [22] introduce the two kinds of risks into the optimal reinsurance and investment problem for an insurer with a power utility preference. Li et al. [23] consider an optimal reinsurance and investment problem under the M-V framework and derive the time-consistent optimal investment strategy. Yao et al. [24] study a dynamic M-V asset allocation problem with stochastic interest rates and inflation rate, where an inflation-indexed bond is not introduced. Liang and Zhao [25] study an optimal M-V efficiency of a family with life insurance under the risks of interest rate and inflation, where the financial market is composed of a risk-free asset (cash), a zero-coupon bond, an inflation-indexed bond and a risky asset (stock). Following the works of Yao et al. [24] and Liang and Zhao [25], Pan and Xiao [26] consider an optimal ALM problem with stochastic interest rates and inflation risks under the M-V framework. While the financial market is composed of a risk-free asset, a zero-coupon bond, an inflation-indexed bond and n risky assets as well as the liability process is described by a geometric Brownian motion rather than a Brownian motion to ensure a definite liability value.

In this paper, based on the work of Pan and Xiao [26], we incorporate the above two important risks of market into a dynamic ALM problem under the expected utility maximization framework. Different from the usual asset allocation problem, the introduction in the dynamic portfolio selection problem of the liability process will be more practical, but causes computational difficulties and requires additional technical skills. To solve the complex optimal control problem, we first derive the extended Hamilton–Jacobi–Bellman (HJB) equation for the optimization problem by using the stochastic dynamic programming approach. Then we obtain the explicit solutions of the optimal investment strategies by solving the extended HJB equation together with the power utility and exponential utility functions. It should be pointed out that the solving process of Eq. (30) is an important reason for obtaining the explicit solutions of the optimal investment strategies. With the explicit solutions of this paper, the numerical analysis can be easily implemented to show the dynamic investment strategies. Moreover, our results provide some efficient ways for investors in characterizing their optimal portfolio strategies of the sophisticated ALM model.

The rest of this paper is organized as follows. Section 2 introduces the financial market and establishes this ALM optimization problem. Section 3 provides the general framework of the optimization problem by applying the stochastic control theory. Section 4 gives the explicit solutions of optimal investment strategies for the power utility and exponential utility functions by using the variable change techniques and partial differential equation (PDE) approach. Section 5 provides numerical examples to show the effects of model parameters on the optimal investment strategies. Section 6 concludes the paper.

2. Problem formulation

In this section, we shall introduce the financial market and establish the ALM optimization problem with stochastic interest rates and inflation risks under the expected utility maximization framework.

2.1. The financial market

Through this paper, $(\Omega, \mathrm{F}, \{\mathrm{F}_t\}_{t\geq 0}, \mathrm{P})$ is a filtered complete probability space on which is defined a standard $\{\mathrm{F}_t\}_{t\geq 0}$-adapted m-dimensional Brownian motion $W(t) = (W_1(t), W_2(t), \cdots, W_m(t))'$, where $(\cdot)'$ represents the transpose of a vector or a matrix, and the Brownian motions $\{W_1(t), W_2(t), \cdots, W_m(t)\}$ are taken independent.

Consider an arbitrage-free financial market where $n+1$ ($n \geq 2$) assets are traded continuously on a finite horizon $[0, T]$. One asset is a nominal risk-free asset (cash), whose price process is denoted by $S_0(t)$ and evolves according to the following ordinary differential equation:

$$\frac{dS_0(t)}{S_0(t)} = R(t)dt, S_0(0) = S_0 > 0, \tag{1}$$

where $R(t)$ is the nominal risk-free interest rate and follows the Hull–White interest rate model

$$dR(t) = a[\theta(t) - R(t)]dt + \sum_{j=1}^{m} \sigma_{Rj} dW_j(t), R(0) = R_0 > 0, \tag{2}$$

where $\theta(t)$ is a deterministic function and denotes the long-run mean of interest rate; a is a positive constant and denotes the degree of mean reversion; $\sigma_R = (\sigma_{R1}, \sigma_{R2}, \cdots, \sigma_{Rm})$ is a constant vector and denotes the volatility of interest rate. To express the common form of the interest rate model, let $\|\sigma_R\| = \sqrt{\sum_{j=1}^m \sigma_{Rj}^2}$ and $W_R(t) = \sum_{j=1}^m \frac{\sigma_{Rj}}{\|\sigma_R\|} W_j(t)$. Then (2) can be rewritten as

$$dR(t) = a[\theta(t) - R(t)]dt + \|\sigma_R\| dW_R(t), R(0) = R_0 > 0. \tag{3}$$

Here we need to point out that the Hull–White interest rate model can take negative values with very small probability in theory. Nevertheless, the use of Hull–White model is very popular in both the academic and business communities for its tractability.

The second asset is a default-free zero-coupon bond (treasury bond). The zero-coupon bond is introduced to reduce the interest rate risk. Let $B(t, T)$ denote the price of a default-free zero-coupon bond which pays one unit of account at maturity T, and $B(t, T)$ has the following exponential affine form of solution (see Boulier et al. [27])

$$B(t,T) = \mathrm{e}^{-\frac{1-\mathrm{e}^{a(t-T)}}{a} R(t) + \int_t^T \{ \frac{\boldsymbol{\sigma_R}\boldsymbol{\sigma_R}'}{2} \frac{[1-\mathrm{e}^{a(s-T)}]^2}{a^2} - \frac{1-\mathrm{e}^{a(s-T)}}{a} [a\theta(s) - \lambda_R(s)\|\boldsymbol{\sigma_R}\|]\} ds},$$

where $\lambda_R(t)$ is the market price of interest rate risk and can be obtained through estimating the price of treasury bonds with different maturities. However, as is studied in Boulier et al. [27], it is quite unrealistic for an investor to find all the zero-coupon bonds in the market. We also introduce a rolling bond (also a kind of default-free zero-coupon bond) in order to keep the maturity T_1 of the bond to be constant, while $T_1 > T$ for preventing arbitrage at time T. Furthermore, applying Ito's lemma to $B(t, T_1)$, the dynamic of $B(t, T_1)$ is

$$\begin{aligned}\frac{dB(t,T_1)}{B(t,T_1)} &= [R(t) - \lambda_R(t)\sigma_B(t)]dt - \sigma_B(t)dW_R(t) \\ &= [R(t) - \lambda_R(t)\sigma_B(t)]dt - \sigma_B(t)\sum_{j=1}^{m} \frac{\sigma_{Rj}}{\|\sigma_R\|} dW_j(t),\end{aligned} \tag{4}$$

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