Chaos, Solitons and Fractals 103 (2017) 513–531

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Analysis of dynamical behaviors of a friction-induced oscillator with switching control law

Jinjun Fan*, Shan Xue, Shuangshuang Li

School of Mathematics and Statistics, Shandong Normal University, Ji'nan 250014, PR China

ARTICLE INFO

Article history:
Received 7 June 2017
Revised 3 July 2017
Accepted 7 July 2017

Keywords:
Friction–induced oscillator
Switching control law
Discontinuous dynamical system
Stick motion
Sliding motion
Passable motion

ABSTRACT

In this paper, the dynamical behaviors of a friction–induced oscillator with switching control law are studied through the flow switching theory of discontinuous dynamical systems. The physical model consists of a mass on the conveyor belt and a spring-damping system with switching control law. Based on the switching control law and the friction between the oscillator and the conveyor belt, multiple domains and discontinuous boundaries are defined. The G–functions are introduced to illustrate the motion switching mechanism and the analytical conditions of the passable motion, stick motion, sliding motion and grazing motion are presented for motion switchability. The switching sets and mapping structures are adopted to describe the complex motions in this discontinuous system. The numerical simulations are also carried out from the analytical conditions and mapping structures in order to better understand the motion switching complexity of this oscillator.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Friction and transversal phenomena exist widely in the production and life. In engineering, the frictional force that exists between the oscillator and the belt, and the setting of the transverse control system are the important factors leading to the discontinuity of the dynamic systems. The research on the friction and transversal phenomena has never stopped. Hartog [1] investigated the periodic motion in the system of forced vibrations and viscous damping in 1930. In 1960, Levitan [2] used the Fourier series to investigate the friction oscillation model with the periodically driven base, and discussed the stability of the periodic motion. In 1979, Hundal [3] studied the response of a single degree of freedom spring-mass system with viscous and Coulomb friction through the closed form analytical solutions. In 1986, Shaw [4] investigated the non–stick periodic motion with dry–friction, and discussed the stability of the motion using Poincare mappings. In 1994, Feeny and Moon [5] experimentally and numerically studied the chaotic dynamics of a harmonically forced spring-mass system with dry friction. In 1999, Virgin and Begley [6] analyzed the grazing bifurcations and basins of attraction in an impact-friction oscillator. In 2000, Dankowicz and Nordmark [7] mainly discussed bifurcations associated with the appearance of stick-slip oscillations based on methods of dynamical system analysis. In 2000, Leine et al. [8] used the Fillipov theory to investigate the bifurcations in nonlinear discontinuous systems. In 2001, Ko et al. [9] studied the dynamics of a friction–induced vibration with or without external disturbance. In 2003, Kim and Perkins [10] improved the superior convergence rates and superior modes of convergence by generalizing the traditional harmonic balance methods, and used an example of a classical single degree-of-freedom model to illustrate this improvement. In 2005, Luo [11–13] developed a local singularity and transversally theory of a flow on the separation boundary from one domain to its adjacent domain, and introduced the real flows and imaginary flows. And he used such a theory to investigate the mapping dynamics and switching conditions of periodic motions for a piecewise linear system under a periodic excitation. In 2006, Luo and Gegg [14] studied the dynamic mechanism of stick and non-stick motion of dry friction vibrators, and analytically predicted the periodic motion with stick and non-stick based on mapping structures. In 2007, the switching dynamics of flow from one domain into another in the periodically driven discontinuous system were presented in Luo and Rapp [15], and the sliding and grazing conditions on the separation boundary were given, and the periodic motions were analytically predicted through different mapping structures. In 2008, Luo [16] introduced the G–functions for discontinuous dynamical systems to investigate the singularity in discontinuous dynamical systems, and discussed the switchability of a flow from a domain to an adjacent one using such G–functions. In 2011 and 2012, Luo [17,18] systematically presented the flow switching theory of discontinuous dynamical systems. Based on this theory, Chen and Fan [19] investigated the

* Corresponding author.
E-mail address: fjj18@126.com (J. Fan).

http://dx.doi.org/10.1016/j.chaos.2017.07.009
0960-0779/© 2017 Elsevier Ltd. All rights reserved.

analytical results of complex motions for a double friction–oscillator in 2016; and Fan et al. [20] studied a friction–induced oscillator with two degree of freedom on a speed–varying traveling belt in 2017. In 2015 and 2017, Zhang and Fu [21–23] studied the periodic motions, stick motions, grazing flows in an inclined impact oscillator and the flow switchability of motions in a horizontal impact pair with dry friction by using the flow switching theory in discontinuous dynamical systems. In 2016, Li and Wu [24] studied the stability of nonlinear differential systems with state–dependent delayed impulses. Li and Song [25] investigated the stabilization of delay systems of delay-dependent impulsive control in 2017. In the same year, Zhang and Li [26] discussed the input–to–state stability of non–linear systems with distributed–delayed impulses.

In 2005, the motions of a friction–induced oscillator with periodic excitation on the constant speed belt was analyzed via the theory of flow switchability for discontinuous dynamical systems in Luo and Gegg [27], and the corresponding numerical simulations were given as well. In 2010, Luo and Rapp [28] studied the sliding and transversal motions on an incline boundary in a periodically driven discontinuous system. The main characteristic of this system was the straight line boundary that was considered as a control law to switch in the phase space. The normal vector field to the separation boundary was given to develop the corresponding transversal conditions. The periodic motions and the corresponding local stability and bifurcation analysis were carried out through the mapping structures. As a matter of fact, the models with friction and switching control law exist extensively in physics, aerospace engineering, and mechanical engineering. For example, the semi active variable stiffness, damping and external excitation control systems are useful in intelligent braking system and seismic control.

In this paper, the flow switchability theory of discontinuous dynamical systems is used to study the complex dynamics of the friction–induced oscillator with switching control law. When the velocity and displacement of the oscillator satisfy the appropriate conditions, that is, the switching control law, the spring stiffness, damping coefficient and external excitation of the oscillator will change. Based on the discontinuities caused by the switching control law and the friction between the oscillator and the conveyor belt, multiple domains and discontinuous boundaries are defined in phase plane. The analytical conditions of passable motion, stick motion, sliding motion and grazing motion are presented through the G–functions. According to the mapping structures and analytical conditions, numerical simulations are given to better understand the complex motions of the friction–induced oscillator with switching control law.

2. Preliminaries

For convenience, the concepts of G–functions and some lemmas are given in the following (see [17,18]).

Definition 1. Consider a dynamic system consisting of N sub-dynamic systems in a universal domain $\Omega \subset R^n$. The universal domain is divided into N accessible sub-domains $\Omega_\alpha (\alpha \in I)$ and the union of inaccessible domain Ω_0. The union of all the accessible sub-domains is $\cup_{\alpha \in I}\Omega_\alpha$ and $\Omega = \cup_{\alpha \in I}\Omega_\alpha \bigcup \Omega_0$ is the universal domain. On the αth open sub-domain Ω_α, there is a C^{r_α}-continuous system ($r_\alpha \geq 1$) in a form of

$$\dot{\mathbf{x}}^{(\alpha)} \equiv \mathbf{F}^{(\alpha)}(\mathbf{x}^{(\alpha)}, t, \mathbf{p}_\alpha) \in R^n, \quad \mathbf{x}^{(\alpha)} = (x_1^{(\alpha)}, x_2^{(\alpha)}, \ldots, x_n^{(\alpha)})^{\mathrm{T}} \in \Omega_\alpha, \tag{1}$$

where time is t and $\dot{\mathbf{x}}^{(\alpha)} = \frac{d\mathbf{x}^{(\alpha)}}{dt}$. In an accessible sub-domain Ω_α, the vector field $\mathbf{F}^{(\alpha)}(\mathbf{x}^{(\alpha)}, t, \mathbf{p}_\alpha)$ with parameter vectors $\mathbf{p}_\alpha = (p_\alpha^{(1)}, p_\alpha^{(2)}, \ldots, p_\alpha^{(l)})^{\mathrm{T}} \in R^l$ is C^{r_α} – continuous ($r_\alpha \geq 1$) in $\mathbf{x}^{(\alpha)} \in \Omega_\alpha$ and for all time t; and the continuous flow in (1) $\mathbf{x}_t^{(\alpha)} = \mathbf{\Phi}^{(\alpha)}(t_0, \mathbf{x}_0^{(\alpha)}, \mathbf{p}_\alpha, t)$ with an initial condition $(t_0, \mathbf{x}_0^{(\alpha)})$ is $C^{r_\alpha+1}$ –continuous for time t.

The flow on the boundary $\partial\Omega_{\alpha\beta}$ of two adjacent domains can be determined by

$$\dot{\mathbf{x}}^{(0)} \equiv \mathbf{F}^{(0)}(\mathbf{x}^{(0)}, t) \text{ with } \varphi_{ij}(\mathbf{x}^{(0)}, t, \lambda) = 0, \tag{2}$$

where

$$\mathbf{x}^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)})^{\mathrm{T}}.$$

With specific initial conditions, one always obtains different flows on $\varphi_{ij}(\mathbf{x}^{(0)}, t, \lambda) = 0$.

Consider a dynamic system (1) in domain $\Omega_\alpha (\alpha \in \{i, j\})$ which has a flow $\mathbf{x}_t^{(\alpha)} = \mathbf{\Phi}^{(\alpha)}(t_0, \mathbf{x}_0^{(\alpha)}, \mathbf{p}_\alpha, t)$ with an initial condition $(t_0, \mathbf{x}_0^{(\alpha)})$ and on the boundary $\partial\Omega_{ij} = \{\mathbf{x} | \varphi_{ij}(\mathbf{x}, t, \lambda) = 0,\ \varphi_{ij}$ is C^r – continuous $(r \geq 1)\} \subset R^{n-1}$, there is an enough smooth flow $\mathbf{x}_t^{(0)} = \mathbf{\Phi}^{(0)}(t_0, \mathbf{x}_0^{(0)}, \lambda, t)$ with an initial condition $(t_0, \mathbf{x}_0^{(0)})$. For an arbitrarily small $\varepsilon > 0$, there are two time intervals $[t-\varepsilon, t)$ and $(t, t+\varepsilon]$ for flow $\mathbf{x}_t^{(\alpha)} (\alpha \in \{i, j\})$, the vector fields $\mathbf{F}^{(\alpha)}(\mathbf{x}_t^{(\alpha)}, t, \mathbf{p}_\alpha)$ and $\mathbf{F}^{(0)}(\mathbf{x}_t^{(0)}, t, \lambda)$ are $C^{r_\alpha}_{[t-\varepsilon, t+\varepsilon]}$-continuous ($r_\alpha \geq k$, k is a positive integer) for time t with

$$\| d^{r_\alpha+1}\mathbf{x}_t^{(\alpha)}/dt^{r_\alpha+1} \| < \infty, \| d^{r_\alpha+1}\mathbf{x}_t^{(0)}/dt^{r_\alpha+1} \| < \infty.$$

Definition 2. The 0th–order G-functions ($G_{\partial\Omega_{ij}}^{(\alpha)}$) of the domain flow $\mathbf{x}_t^{(\alpha)}$ to the boundary flow $\mathbf{x}_t^{(0)}$ on the boundary in the normal direction of the boundary $\partial\Omega_{ij}$ are defined as

$$\begin{aligned} &G_{\partial\Omega_{ij}}^{(\alpha)}(\mathbf{x}_t^{(0)}, t_\pm, \mathbf{x}_{t_\pm}^{(\alpha)}, \mathbf{p}_\alpha, \lambda) \\ &\quad \equiv G_{\partial\Omega_{ij}}^{(0,\alpha)}(\mathbf{x}_t^{(0)}, t_\pm, \mathbf{x}_{t_\pm}^{(\alpha)}, \mathbf{p}_\alpha, \lambda) \\ &\quad = D_{\mathbf{x}_t^{(0)}}\, {}^t\mathbf{n}_{\partial\Omega_{ij}}^{\mathrm{T}} \cdot (\mathbf{x}_{t_\pm}^{(\alpha)} - \mathbf{x}_t^{(0)}) + {}^t\mathbf{n}_{\partial\Omega_{ij}}^{\mathrm{T}} \\ &\qquad \cdot [\mathbf{F}^{(\alpha)}(\mathbf{x}_{t_\pm}^{(\alpha)}, t_\pm, \mathbf{p}_\alpha) - \mathbf{F}^{(0)}(\mathbf{x}_t^{(0)}, t, \lambda)], \end{aligned} \tag{3}$$

where ${}^t\mathbf{n}_{\partial\Omega_{ij}}$ is the normal vector of the boundary surface $\partial\Omega_{ij}$ and $t_\pm = t \pm 0$ is to represent the quantity in the domain rather than on the boundary.

Definition 3. The kth-order G-functions ($G_{\partial\Omega_{ij}}^{(k,\alpha)}$) of the domain flow $\mathbf{x}_t^{(\alpha)}$ to the boundary flow $\mathbf{x}_t^{(0)}$ on the boundary in the normal direction of $\partial\Omega_{ij}$ are defined as

$$\begin{aligned} &G_{\partial\Omega_{ij}}^{(k,\alpha)}(\mathbf{x}_t^{(0)}, t_\pm, \mathbf{x}_{t_\pm}^{(\alpha)}, \mathbf{p}_\alpha, \lambda) \\ &\quad = \Sigma_{s=1}^{k+1} C_{k+1}^s D_{\mathbf{x}_t^{(0)}}^{k+1-s}\ {}^t\mathbf{n}_{\partial\Omega_{ij}}^{\mathrm{T}} \\ &\qquad \cdot [D_{\mathbf{x}_t^{(\alpha)}}^{s-1}\mathbf{F}(\mathbf{x}_{t\pm}^{(\alpha)}, t_\pm, \mathbf{p}_\alpha) - D_{\mathbf{x}_t^{(0)}}^{s-1}\mathbf{F}^{(0)}(\mathbf{x}_t^{(0)}, t, \lambda)] \\ &\qquad + D_{\mathbf{x}_t^{(0)}}\, {}^t\mathbf{n}_{\partial\Omega_{ij}}^{\mathrm{T}} \cdot (\mathbf{x}_{t_\pm}^{(\alpha)} - \mathbf{x}_t^{(0)}), \end{aligned} \tag{4}$$

where k is a positive integer.

In above definitions, the total derivative is

$$D_{\mathbf{x}_t^{(0)}}(\cdot) \equiv \frac{\partial(\cdot)}{\partial\mathbf{x}_t^{(0)}} \cdot \dot{\mathbf{x}}_t^{(0)} + \frac{\partial(\cdot)}{\partial t},$$

and the normal vector of the boundary surface $\partial\Omega_{ij}$ at point $\mathbf{x}_t^{(0)}$ is given by

$$\begin{aligned} &{}^t\mathbf{n}_{\partial\Omega_{ij}}(\mathbf{x}_t^{(0)}, t, \lambda) = \nabla\varphi_{ij}(\mathbf{x}_t^{(0)}, t, \lambda) \\ &\quad = \left(\frac{\partial\varphi_{ij}}{\partial x_1^{(0)}}, \frac{\partial\varphi_{ij}}{\partial x_2^{(0)}}, \cdots, \frac{\partial\varphi_{ij}}{\partial x_n^{(0)}}\right)^T \Big|_{(t,\, \mathbf{x}_t^{(0)})}. \end{aligned}$$

Considering the flow $\mathbf{x}_t^{(\alpha)}$ contacts with the boundary $\mathbf{x}_m \in \partial\Omega_{ij}$ at the time t_m, that is $\mathbf{x}_{t_{m\pm}}^{(\alpha)} = \mathbf{x}_m = \mathbf{x}_{t_m}^{(0)}$, the 0th–order G–functions

Download English Version:

https://daneshyari.com/en/article/5499594

Download Persian Version:

https://daneshyari.com/article/5499594

Daneshyari.com