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Multistability and organization of periodicity in a Van der Pol–Duffing oscillator

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ABSTRACT

We investigate the dynamics of a Van der Pol–Duffing forced oscillator, which is modelled by a five-parameter second order nonautonomous nonlinear ordinary differential equation. Firstly we fix three of these parameters, and investigate the dynamics of this system by varying the other two, namely the amplitude and the angular frequency of the external forcing. We also investigate the existence of different attractors, periodic, quasiperiodic, and chaotic. Finally, we investigate the occurrence of multistability in the considered Van der Pol–Duffing forced oscillator, for some fixed sets of parameters.

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1. Introduction

In this paper we investigate numerically the nonlinear dynamics of a Van der Pol–Duffing forced oscillator, whose general mathematical form is given by [1]

$$\ddot{x} - \mu(1 - x^2)\dot{x} + \omega_0^2 x + \alpha x^3 = f\cos(\omega t), \tag{1}$$

where f and ω are respectively the amplitude and the angular frequency of the external forcing, and μ, ω_0, and α are control parameters related to the respective unforced system. As can be seen in Eq. (1), the dissipation is modelled by a nonlinear velocity dependent term given by $G(x, \dot{x}) = -\mu(1 - x^2)\dot{x}$, the damping force is derived from a potential function $V(x) = \omega_0^2 x^2/2 + \alpha x^4/4$ which represents a double-well potential when $\omega_0 < 0$, $\alpha > 0$, and $F(t) = f\cos(\omega t)$ is a sinusoidal external forcing.

It is straightforward to show that Eq. (1) reduces to

$$\ddot{x} + \omega_0^2 x + \alpha x^3 = f\cos(\omega t), \tag{2}$$

when $\mu = 0$, which may be considered as one of the simplest versions of the Duffing equation [2], investigated originally in the context of vibrations. Alternatively, if we consider $\alpha = 0$ Eq. (1) becomes

$$\ddot{x} - \mu(1 - x^2)\dot{x} + \omega_0^2 x = f\cos(\omega t), \tag{3}$$

which is the Van der Pol equation [3] with an external forcing. Although the Van der Pol oscillator has been originally proposed to model electrical circuits, it was also used as a mathematical model in other fields of the knowledge, for instance in physical [4] and biological [5] systems.

Van der Pol-Duffing oscillators forced in different ways have a wide usage in many fields, with consequent application to model the most varied nonlinear processes. Some few examples consider the modeling of the stochastic response of a vibroimpact system under additive colored noise excitation [6] and optical bistability in a dispersive medium [7]. Additionally, couplings of Van der Pol–Duffing oscillators are useful to model electroencephalogram signals [8] and microelectromechanical systems resonators [9], which have significant use as sensors, biomedical implants, and wireless communication devices.

Because of this wide range of applications, the nonlinear dynamics of the Van der Pol–Duffing oscillator has been investigated widely in these years. More recently an analytical investigation, concerning the nonlinear dynamics of the Van der Pol–Duffing forced oscillator modeled by Eq. (1) was reported by Cui and collaborators [1]. In this reference, based on the homotopy analysis method, the authors showed that unstable periodic solutions can be obtained for the oscillator (1), when the set of parameters $\mu = 0.1$, $\omega_0 = \alpha = f = 1$, and $\omega = 2$ is considered. Also recently, another Van der Pol–Duffing forced oscillator, obtained by making $\omega_0 = 0$, $\alpha = 1$, and $f = 1$ was investigated considering ω as a control parameter [10]. This time, multistability, the coexistence of attractors for a given set of parameters, was detected. Coupled

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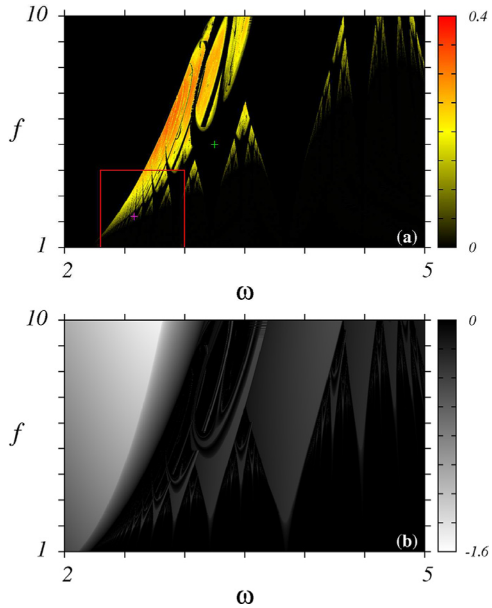

Fig. 1. Regions and colors in the (ω, f) parameter plane of system (1), according to estimates of the Lyapunov exponents (see the text). (a) Considering the largest Lyapunov exponent. (b) Considering the second largest Lyapunov exponent. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

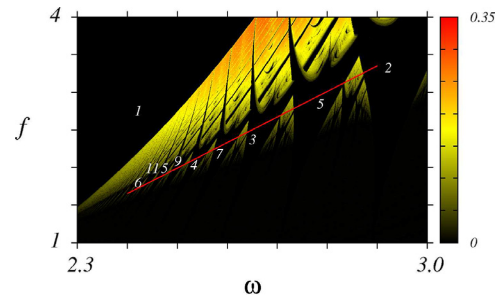

Fig. 2. Enlargement of the region inside the box in Fig. 1(a). With regard to the straight line in red, see the text. Numbers refer to periods. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Van der Pol-Duffing forced oscillators were also investigated in Ref. [10].

Our contribution to advancing the knowledge of the Van der Pol–Duffing forced oscillator, involves an investigation concerning the organization of different dynamical behaviors, namely chaos, quasiperiodicity, and periodicity, in the (ω, f) parameter plane of system (1). For this purpose, the parameters related to the unforced system in Eq. (1) are kept fixed as $\mu = 0.5$, $\omega_0 = \alpha = 1$, and (ω, f) parameter plane plots displaying dynamical behaviors are constructed. The Lyapunov exponents spectrum is used to numerically characterize the dynamics of each point in these parameter planes. Additional contribution of our work is related to studies on multistability in system (1), with the consequent construction of basins of attraction for suitable places in the (ω, f) parameter planes mentioned above.

The paper is organized as follows. In Section 2 we present numerical results related to the organization of the dynamics in the (ω, f) parameter plane of system (1). Basins of attraction of system (1) are considered in Section 3. Finally, concluding remarks are given in Section 4.

2. The (ω, f) parameter plane of the Van der Pol–Duffing oscillator

The diagrams in Fig. 1 show a global vision of the dynamical behaviors present in the (ω, f) parameter plane of system (1), when $\mu = 0.5$, $\omega_0 = \alpha = 1$. Estimates to the values of the largest and the second largest Lyapunov exponent (LLE) are associated to colors, respectively in diagrams (a) and (b), according to the color scale at right hand side in each of them. Hence, a same region that appears painted in black in both diagrams of Fig. 1 is corresponding to (ω, f) parameters for which the related attractors in the phase-space are quasiperiodic, since the LLE and the second LLE are both equal to zero for points in this region. This quasiperiodic region is that at the bottom in the diagrams of Fig. 1. On the other hand, a same region that appears painted in black in Fig. 1(a) and painted in grey in Fig. 1(b), is corresponding to (ω, f) parameters that generate periodic attractors in the phase-space. In this case, the LLE is equal to zero, and the second LLE is less than zero. To complete the interpretation of the colors in the diagrams of the Fig. 1, we need to talk about a same region painted with a gradient from yellow to red in Fig. 1(a), and painted in black in Fig. 1(b). Such region is corresponding to (ω, f) parameters for which the related attractors in the phase-space are chaotic. Points in this region are characterized by a LLE greater than zero and a second LLE equal to zero. This procedure, which uses the Lyapunov exponents spectra, more specifically the two LLE, to numerically characterize the dynamics of points in parameter planes of a three-dimensional continuous-time dissipative dynamical system, is based in a method explained in details in Ref. [11].

The (ω, f) diagrams (a) and (b) in Fig. 1, were obtained by computing respectively the LLE and the second LLE, in a grid of $10^3 \times 10^3$ parameters. System (1) was integrated by using a fourth-order Runge-Kutta algorithm, with a fixed time step equal to 10^{-3}, being considered 1×10^6 integration steps. Thus, the average involved in the computation of the Lyapunov exponents spectrum, was performed by considering 1×10^6 points of the respective trajectory in the phase-space, for each one of the 1×10^6 pairs of parameters. Each diagram in Fig. 1 was constructed from an initial condition within the basin of attraction of system (1), for a fixed pair of parameters, in fact the two lowest $\omega = 2$ and $f = 1$. The variables [12] at the end of the integration for this pair of parameters were used to initialize the integration to the next pair, and so forth up to the highest value of both parameters, $\omega = 5$ and $f = 10$, be achieved.

As we can see in diagrams of Fig. 1, for a small amplitude of the forcing f, near to $f = 1$, the Van der Pol–Duffing forced oscillator is quasiperiodic in a large range of the investigated angular frequency, namely for $2.2 < \omega < 5$ approximately. Born in this quasiperiodic region of the (ω, f) parameter plane, and spreading by the chaotic region, we observe organized periodic structures, similar to the Arnold tongues present in the circle map [13], detected and measured in experiments involving wave-particle interaction [14]. These organized periodic structures are better seen in the amplification of the boxed region of Fig. 1(a), which is shown in Fig. 2. Number identifying each periodic window in Fig. 2 is related with the period of the respective structure, being period here assumed as the number of local maxima of the variable x [12], namely x_m, in one complete orbit on the $(x, \dot{x})$ phase-space attractor. Such windows are usually displayed in conventional bifurcation diagrams like that shown in Fig. 3, which helps us to determine the periods of some structures, like that numbered in Fig. 2.

The bifurcation diagram in Fig. 3 considers 10^3 points along the red straight line $f = 3.4\omega - 6.51$, that crosses the periodic structures in Fig. 2. It was obtained by plotting the local maxima values of the variable x, as a function of the parameter ω. As before in obtaining Figs. 1 and 2, integrations were performed by using

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