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Influence of a nonlinear coupling on the supratransmission effect in modified sine-Gordon and Klein–Gordon lattices

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ABSTRACT

In this paper, we analyze the conditions leading to the nonlinear supratransmission phenomenon in two different models: a modified fifth order Klein–Gordon system and a modified sine-Gordon system. The modified models considered here are those with mixed coupling, the pure linear coupling being associated with a nonlinear coupling. Especially, we numerically quantify the influence of the nonlinear coupling coefficient on the threshold amplitude which triggers the nonlinear supratransmission phenomenon. Our main result shows that, in both models, when the nonlinear coupling coefficient increases, the threshold amplitude triggering the nonlinear supratransmission phenomenon decreases.

1. Introduction

Contrary to linear media, it is possible to transmit energy in nonlinear propagation systems outside their linear frequency band. As an example, it has been shown recently that nonlinear split-ring resonator (SRRs) transmission lines may transmit significant power by means of nonlinearity-induced frequency bands [1]. On the other hand, it is also well known that, when a nonlinear and discrete transmission line is excited with a frequency in the gap, it may transmit energy if the input signal amplitude exceeds a particular threshold. This phenomenon, called nonlinear supratransmission, was introduced by Geniet and Léon [2,3] when they observed a sudden increase in amplitude of the signal transmitted in a nonlinear chain excited with a frequency in the gap. Subsequently, several theoretical and/or numerical studies have also demonstrated the existence of this phenomenon in a wide number of systems. As an exemple, numerical simulations and theoretical studies have shown that the classical sine-Gordon model may transmit energy in the gap via the nonlinear supratransmission phenomenon [4,5] and that its nonlinearity enables a bistable behavior [6]. Other studies on the classical sine-Gordon model have shown that noise can also contribute to produce nonlinear modes via the nonlinear supratransmission phenomenon [7]. The one dimensional case has been addressed, and an extension to a 2-dimensional (2D) system has been also reported [8]. These studies led to some applications, like the propagation of binary signals in semi-infinite mechanical chains of coupled oscillators [9,10], in the bounded β-Fermi–Pasta–Ulam lattice [11], and on the transmission of binary signals in the continuous Frenkel–Kontorova medium [12]. The study of nonlinear supratransmission was also conducted in many others systems like nonlinear transmission lines [13–15], waveguide arrays [16–19], Bragg media [20], pendula chains [21], optical lattices [22], birefringent media with quadratic nonlinearity [23], Josephson junctions [24–29] to cite but a few. All of these studies were made while considering only systems with pure linear coupling, but other ones were conducted on systems with mixed coupling [30–32]. However, these latest works have not shown the influence of the nonlinear coupling on the nonlinear supratransmission phenomenon. In this paper, we propose to numerically study how the addition of the nonlinear coupling affects supratransmission in the fifth order Klein–Gordon system [3,33] and in the sine-Gordon medium.

The paper is organized as follows. In the next section, we present the models. The following section is then devoted to the study of the fifth order Klein–Gordon system, in the classical case, that is without nonlinear coupling and in the case including an additive nonlinear coupling. Section 4 discusses the case of the sine-Gordon model including the nonlinear coupling and Section 5 concludes the paper.

2. Models description

We consider a network of N particles whose displacement U_n for the nth particle obeys to :

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$$\frac{d^2U_n}{dt^2} = -\gamma\frac{dU_n}{dt} + \omega_L^2(U_{n+1} - 2U_n + U_{n-1}) + \omega_{NL}^2\left[(U_{n+1} - U_n)^3 + (U_{n-1} - U_n)^3\right] - \omega_0^2 f(U_n), \quad (1)$$

where the first cell is sinusoidally driven by the excitation $U_0(t) = A\sin(\omega t)$.

The dissipation parameter γ is taken constant at any time for all oscillators constituting the nonlinear lattice and it will be set to 0.01 in all our study. Moreover, we consider a network of $N = 600$ particles.

In Eq. (1), for the Klein–Gordon system, the nonlinearity $f(U_n)$ obeys to

$$f(U_n) = U_n - \frac{U_n^3}{3!} + \frac{U_n^5}{5!}, \quad (2)$$

while for the sine-Gordon model, the nonlinearity is defined by

$$f(U_n) = \sin(U_n). \quad (3)$$

Note that, in both cases, in the linear approximation, the function $f(U_n)$ reduces to

$$f(U_n) = U_n. \quad (4)$$

The theoretical expressions of the dispersion relation and damping can be deduced by considering the following profile in Eq. (1)

$$U_n(t) = U_0 e^{j(\beta n - \omega t)} + U_0 e^{-j(\beta n - \omega t)}, \quad (5)$$

where β is a complex wave number that can be written under the form

$$\beta(\omega) = k(\omega) + j\alpha(\omega). \quad (6)$$

In expression (6), k corresponds to the wave number while α denotes the damping. Substituting the linear plane wave defined by expression (5) in Eq. (1) with the linear approximation (4) leads to

$$\frac{\omega_0^2 - \omega^2}{2\omega_L^2} = j\frac{\omega\gamma}{2\omega_L^2} + \cos(k)\cosh(\alpha) - j\sin(k)\sinh(\alpha) - 1. \quad (7)$$

By identifying the real and imaginary parts of Eq. (7), we obtain

$$\begin{cases} 1 + \dfrac{\omega_0^2 - \omega^2}{2\omega_L^2} = \cos(k)\cosh(\alpha) \\ \dfrac{\omega\gamma}{2\omega_L^2} = \sin(k)\sinh(\alpha) \end{cases}. \quad (8)$$

Restricting our study to the case of weak values of α (either $\alpha \to 0$), we can consider the following approximations $\cosh(\alpha) \approx 1$ and $\sinh(\alpha) \approx \alpha$, which leads to the theoretical expression of the dispersion relation:

$$\omega^2 = \omega_0^2 + 2\omega_L^2(1 - \cos k), \quad (9)$$

where ω is the angular frequency of linear waves and k their wave number. The corresponding linear spectrum exhibits a band-pass behavior of bandwidth $[\omega_0; \omega_c]$ with a high cut-off frequency $\omega_c = \sqrt{\omega_0^2 + 4\omega_L^2}$ as presented in Fig. 1.

By considering $\sinh(\alpha) \simeq \alpha$ in Eq. (8) and by using the dispersion relation (9), we also have the following theoretical expression of the damping coefficient α:

$$\alpha(\omega) = \frac{\omega\,\gamma}{2\omega_L^2\sin\left[\arccos\left(1 + \frac{\omega_0^2 - \omega^2}{2\omega_L^2}\right)\right]}. \quad (10)$$

The damping profile is presented in Fig. 2 for the particular case where $\omega_0^2 = 1$ and $\omega_L^2 = 1$. The solid line represents the theoretical expression given in Eq. (10) and the symbols "x" are deduced by numerical simulation of the model (1) driven by the excitation

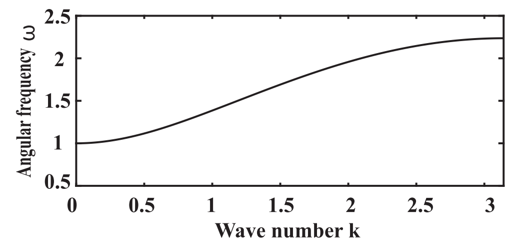

Fig. 1. Dispersion relation profile of both Klein–Gordon and sine-Gordon models defined by Eq. (9). Parameters: $\omega_0^2 = 1$, $\omega_L^2 = 1$.

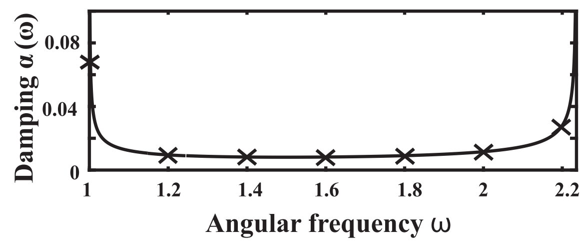

Fig. 2. Damping coefficient α versus the angular frequency ω. The theoretical law (11) is plotted in solid line, while the crosses are deduced from numerical simulation of the model defined by Eq. (1). Parameters : $\omega_0^2 = 1$, $\omega_L^2 = 1$.

$U_0(t) = A\sin\omega t$. Indeed, for each frequency ω inside $[\omega_0; \omega_c]$, that is inside the system bandwidth, we have determined the maximum value reached by $U_n(t)$ for each cell n. Next, to obtain the damping $\alpha(\omega)$ corresponding to the angular frequency ω, we have considered the following decaying law

$$\max[U_n(t)] = Ae^{-\alpha n}. \quad (11)$$

The damping coefficient α was then identified by means of Eq. (11) with a least square method. The evolution of damping versus the angular frequency ω is presented in Fig. 2, where the theoretical law (11) matches the numerical predictions with a perfect agreement.

When the system is driven with an excitation of weak amplitude in the gap ($\omega < \omega_0$), the medium supports an evanescent wave whose profile was mathematically predicted by Geniet and Léon in the linear regime and in the absence of nonlinear coupling ($\omega_{NL}^2 = 0$) [3]. In this linear limit and for an angular frequency inside the gap, the evolution of the cell n obeys to:

$$U_n(t) = A\sin(\omega t)\exp(-\lambda n). \quad (12)$$

In expression (12), λ is determined considering that if $\omega < \omega_0$, then the wave number k is imaginary, that is $k = j\lambda$. By substituting $k = j\lambda$ in the dispersion relation (9), we obtain

$$\lambda = \operatorname{arccosh}\left(1 + \frac{\omega_0^2 - \omega^2}{2\omega_L^2}\right). \quad (13)$$

To extend the validity of the theoretical evanescent profile (12) to the case where the nonlinear coupling is taken into account, we have performed numerical simulations of the modified sine-Gordon model (1) with the following Dirichlet conditions [7] :

$$\begin{cases} U_0(t) = A\sin(\omega t) \\ U_n(0) = 0 \end{cases}, \quad \dot{U}_n(0) = A\omega\exp(-\lambda n). \quad (14)$$

In this paper, all numerical simulations were performed by using a fourth order Runge Kutta algorithm with integration time step $dt = 0.01$ to solve (1) with the Dirichlet conditions (14). First, for different nonlinear coupling values, we have compared the theoretical evanescent profile (12) with the state of the modified sine-Gordon lattice obtained numerically at given time t, namely $t = 30$.