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Review

A memristive chaotic system with heart-shaped attractors and its implementation

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ABSTRACT

As a controllable nonlinear element, memristor is easy to produce the chaotic signal. Most of the current researchers focus on the nonlinear characteristics of the memristor, however, its ability to control and adjust chaotic systems is often neglected. Therefore, a memristive chaotic system is introduced to generate a kind of heart-shaped attractors in this paper. To further understand the complex dynamics of the system, several basic dynamical behavior of the new chaotic system, such as dissipation and the stability of the equilibrium point is investigated. Some basic properties such as Poincaré-map, Lyapunov index and bifurcation diagram are presented, either analytically or numerically. In addition, the influence of parameters on the system's dynamic behavior is analyzed. Finally, an analog implementation based on PSPICE simulation is also designed. The obtained results clearly show this chaotic system has rich nonlinear characteristics. Some interesting conclusions can be drawn that memristors bring the following effects on chaotic systems: (a) when the polarity of the memristor is changed, a mirror image of the chaotic attractors will appeared in the system; (b) along with the proper choose of the memristor parameters, the chaotic motion of system will be suppressed and enhanced, which makes the system can be applied to the practice on either generating chaos signal or suppressing chaotic interference.

1. Introduction

Chaos is descripted as a kind of random change or an irregular movement occurring in a deterministic system, and chaotic state is considered as the fourth state besides equilibrium state, periodic state and quasi periodic state. An early development in the history of Chaotic Systems dates back to 1963 in which an American meteorologist Lorenz try to bring forward system equations to simulate the weather changes. The Lorenz system, as the first chaotic model, revealed the complex and fundamental behaviors of the nonlinear dynamical systems [1]. On the basis of Lorenz system, Professor Chen used method of feedback control to construct a new three-dimensional autonomous chaotic system [2]. And then the concept of the generalized Lorenz system was then extended by Lü et al., and a class of generalized Lorenz-like systems was discussed [3]. Recent years have been increased attention being given to the theory and application research of chaos [4–6]. Numerous new chaotic systems are found and constructed constantly [7–12], and it can make people have a deeper understanding of the chaotic phenomenon, enrich and improve the research content of chaos theory. Chaotic motion and related theories will have a broad application prospect in signal processing, secure communication, electronic engineering, biology and other fields.

As an adjustable nonlinear element, memristor [13–16] is easy to be used as nonlinear part in chaos generator. Furthermore, it has the characteristics of small volume and low power consumption, and memristor has become an ideal choice for nonlinear elements in the chaotic circuit, therefore, various kinds of chaotic systems based on memristors have received great attention from the researchers [17]. Itoh and Chua [18] adopted a flux-controlled piecewise linear memristor model to replace the Chua's diode in Chua's chaotic circuit. As a result, it not only complemented the first memristor-based chaotic systems but also expanded people's understanding of memristor-based chaotic system. Subsequently, researchers realized a chaotic generator based on a PWL memristor by using an active PWL memristor instead of Chua's diode in Chua's circuit [19]. Similarly, Muthuswamy [20] used basic electronic devices such as operational amplifier and multiplier to achieve an equivalent circuit of the memristor, and the smooth memristor model is utilized to replace Chua's diode, then some other new memristor chaotic circuits have been realized. These

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chaotic circuits under certain conditions of circuit parameters can generate different shapes of chaotic attractors. Bao et al. made a further study of the memristor chaotic circuit, and a typical Chua's chaotic circuit with a cubic memristor was further studied in [21]. They implemented a series of new Chua's memristor chaotic circuit and obtained a class of corresponding Chua's double scroll chaotic attractors. Stork [22] et al. devoted to developing a memristor-based feedback system, and the memristor model in this paper has the characteristics of piecewise linear, and then the chaotic attractor is generated. By using the nonlinear characteristics of the memristor, Wang et al. [23,24] successfully deduced a magnetron memristor model, applied it to the proposed system, and the memristor-based chaotic system have been obtained. Using the natural nonlinearities of memristors to produce chaotic signal, which is presently a hot research topic. However, the current researchers mostly focus on the nonlinear characteristics of the memristor, and often ignore its resistance variability. Therefore, this paper aimed at building a bridge between memristor parameters and chaotic systems.

The rest of the paper is organized as following. Section 2 presents the new three-dimensional autonomous chaotic system. In this system, the magnetron memristor model is combined with the chaotic system, which contains two cross product terms and one square term. The basic dynamic behaviors including symmetry and dissipation, system equilibria, the power spectra of system and Poincaré section are investigated respectively in this section. The memristor's regulation function in the formation and control of chaotic systems is introduced in Section 2.5.1. And the impacts of different parameters on the dynamic behaviors of the system are analyzed in detail by using bifurcation diagram and Lyapunov exponent spectrum. In Section 3, an electronic circuit of the chaotic system is designed, and the chaotic attractor is observed in the simulation experiment. The results show that under the same chaotic behavior expectations, the simulation results are identical with the theoretical analysis results, which further verifies the physical existence of the chaotic system. Finally, conclusions are drawn in Section 4.

2. A new memristor-based chaotic system and its basic properties

2.1. The new three-dimensional chaotic system model

The proposed memristor-based chaotic system equations:

$$\begin{cases} \dot{x} = -ay \\ \dot{y} = bx + cy - yz + gf(-|x|) \\ \dot{z} = -dxy - ez + y^2 \end{cases} \tag{1}$$

where, $x, y, z \in \mathbb{R}$ are the state variables, a, b, c, d, e, g are real constants, and the term $f(-|x|)$ refers to the charge of the memristor [23] as shown in the following formula

$$f(x) = \begin{cases} \dfrac{x - n_3}{R_{OFF}} & x < n_1 \\ \dfrac{\sqrt{2kx + M^2(0)} - M(0)}{k} & n_1 < x < n_2 \\ \dfrac{x - n_4}{R_{ON}} & x \geq n_2 \end{cases} \tag{2}$$

with

$$\begin{cases} n_1 = \dfrac{R_{OFF}^2 - M^2(0)}{2k} \\ n_2 = \dfrac{R_{ON}^2 - M^2(0)}{2k} \\ n_3 = \dfrac{[R_{OFF} - M(0)]^2}{2k} \\ n_4 = \dfrac{[R_{ON} - M(0)]^2}{2k} \end{cases} \tag{3}$$

where x is the magnetic flux which is input to the memristor, and the parameters for the memristor model are set as: $R_{ON} = 100\Omega$, $R_{OFF} = 20k\Omega$, $M(0) = 16k\Omega$, $D = 10nm$, $\mu_v = 10^{-14} m^2 s^{-1} V^{-1}$ and k is a constant,

$$k = \frac{(R_{ON} - R_{OFF})\mu_V R_{ON}}{D^2} \tag{4}$$

The system (1) includes four nonlinear terms and four linear terms. When set the parameters value as $a = 12$, $b = 2.2$, $c = 0.1$, $d = 22$, $e = 0.5$ and $g = 10^4$, the system is generating a typical chaotic attractor, and the phase trajectory is shown in Fig. 1(a)–(d). From Fig. 1, we can find that the chaotic attractor of system (1) is different from what we have found before.

2.2. Theoretical analysis of the dynamic characteristics

2.2.1. Dissipation and the existence of the attractor

According to Eq. (1), the divergence of the system is

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = c - e \tag{5}$$

where V indicates the phase volume, as it meets $\nabla V = c - e = -0.4 < 0$, the state change of the system is bounded, therefore system (1) is dissipative. And it converges in the index of the following form

$$\frac{dV}{dt} = e^{(c-e)t} \tag{6}$$

That is, in the dynamic system (1), the time rate of change of phase volume is $e^{(c\text{-}e)t}$, a volume element V_0 is apparently contracted by the flow into a volume element $V_0 e^{(c\text{-}e)t}$ at time t. It means that each volume containing the trajectories of system (1) will shrinks to zero when $t \to \infty$ with the exponential rate $V_0 e^{(c\text{-}e)t}$, as illustrate in Fig. 2. Thus, all the orbits of the system (1) are finally confined to a specific subset of zero volume, and the asymptotic motion of system (1) will be fixed on an attractor of the system [11,25,26].

2.2.2. Equilibria and stability

In order to solve the equilibrium of the proposed system, make the Eq. (1) as

$$\begin{cases} -ay = 0 \\ bx + cy - yz + gf(-|x|) = 0 \\ -dxy - ez + y^2 = 0 \end{cases} \tag{7}$$

Based on Eq. (7), we obtain only one equilibrium point of the system, namely, $S_0 = (0, 0, 0)$. In general, the number of equilibrium points is not equal to the order of the system, that is, the number of the equilibrium points is independent of the order of the system. To linearize the system shown by Eq. (1) at the equilibrium point S_0, the corresponding Jacobian matrix is expressed by

$$A_0 = \begin{bmatrix} 0 & -a & 0 \\ B & c & 0 \\ 0 & 0 & -e \end{bmatrix} \tag{8}$$