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Probabilistic behavior analysis of a sandwiched buckled beam under Gaussian white noise with energy harvesting perspectives

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ABSTRACT

In this paper, a sandwiched buckled beam with axial compressive force under Gaussian white noise is considered as a piezoelectric energy harvester. A stochastic averaging method is proposed to analytically predict the system's response, the stability and the estimation of system's reliability. By using the generalized harmonic transformation, the Itô differential equations with respect to the mechanical and electrical amplitude are derived through this technique. From these differential equations, we construct the Fokker–Plank–Kolmogorov equation for the electrical and mechanical subsystem where the solution of each equation in the stationary state is a probability density. The mean first passage time (*MFPT*) is numerically provided in order to study the attractor stability(stable equilibrium point observed in the effective potential) which give rise to the noise-enhanced stability(*NES*) phenomenon. The mean square response and voltage are obtained for different white noise intensities and others system parameters. The effects of linear damping and noise intensity on the mean square voltage are investigated. We notice that harvested energy can be enhanced by suitable choice of noise intensity and others system parameters. In additional, by combining the random signal with harmonic excitation, the stochastic resonance(*SR*) phenomenon is observed via the mean residence time(*TMR*) which give rise to the large amplitude of vibrations and consequently, an optimization of harvested energy. The agreements between the analytical method and those obtained numerically validate the effectiveness of analytical investigations.

1. Introduction

The fluctuating quantities considered as excitations in many mechanical systems cannot always be adequately modeled by deterministic time functions. In fact, there are several natural phenomena that vary in a random manner due to the effect of many unknown factors, that fluctuate randomly over a wide band of frequencies and have to be considered as stochastic functions of time, defined only in probabilistic terms [1]. Dynamic systems in such environments are subject to stochastic excitations. Then, in order to examine their response and stability, a probabilistic approach employing the theory of stochastic bifurcation is essential. If a system could change from one equilibrium state to another, even small fluctuations of parameters can have important effects. Such effects, when happening in the presence of small stochastic excitations are called stochastic bifurcation. Stochastic bifurcation is a very interesting nonlinear phenomenon and its phenomenological aspect, called p-bifurcation has been intensively studied since 1963 [2–4], analytically and numerically in many systems [5]. For safety, reliability and economic reasons, the nonlinearities of many dynamic energy harvesting systems under environmental and other forces that are treated as random disturbances must be taken into account in the design procedures [6]. Due to the fact that the influence of stochastic bifurcation becomes crucial in the choices made by the system in the course of its evolution between the numerous basins of attraction, or dissipative structures, to which bifurcations give rise [1], its study in the field of energy harvesting could give rise to a better harvesters understanding and control. A particularly simple demonstration of this feature can be achieved by finding the so-called probability structure of system's response based on the Fokker–Plank–Kolmogorov equation (FPK) [7].

Many monographs and reviews have been published to present the state of recent research in energy harvesting technologies [8], piezoelectric energy harvesting [9], vibration energy harvesting via bistable systems [10], the role of nonlinearities in vibratory energy

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harvesting [11]. In the latter [11], authors reported that contrary to one's intuition, the performance metrics of nonlinear energy harvesting from vibratory source, are much easier to define when the excitation is random in nature, and that in such case, the useful statistical averages of the input and output can be easily defined. Many recent researches have then been focused on the experimental and theoretical analysis of different size devices used to harvest energy from ambient random vibrations. Jin et al. [12] found a Semi-analytical solution of random response for nonlinear vibration energy harvesters; the energy-dependent frequency have been obtained through an averaging process. Zhao et al. [13] studied the deterministic and band-limited stochastic energy harvesting from uniaxial excitation of a multilayer piezoelectric stack; they found that a quadratic relation between maximum power and pressure is observed for bandlimited stochastic excitation. Borowiec et al. [14], studied the effect of noise on energy harvesting in a beam with stopper. They showed how the noise component of the excitation influences the stability of the solution. The idea of association of two piezoelectric harvesters to produce more efficient electric power generation, considering the combination of random and stochastic excitations have been studied by Litak [15]. He found a substantial effect of noise on the system dynamics and power generation.

Although as now, energy harvesting from nonlinear oscillators subjected to random excitation has been the subject of investigation by a number of researchers [16–19], the study of stochastic p-bifurcation in harvester models just started. Recently Kumar et al. [7], demonstrated the agreement between their results from the Fokker–Planck approach and Monte Carlo simulation; and a good agreement when compared their simulation with analytic results obtained by Daqaq [20,21].

A particular interest here is the work of Cottone et al. [22], where a detailed study of a piezoelectric buckled beam for random vibration energy harvesting was conducted. In the preceding mentioned work, the authors noticed that nonlinear bistable configuration of the oscillator induced by buckling phenomenon, has been proven to show higher global performances when excited by random vibrations, with power gains up to more than a factor of ten compared to the unbuckled state.

Among the analytical methods known in the literature for solving nonlinear dynamical systems who have submitted to a random excitation and nonlinear damping, stochastic averaging method [23] is a powerful approximation technique for prediction response, stability and estimation of reliability of linear and nonlinear oscillators. This technique has been extensively used in theoretical investigation and engineering application of random vibration. The success of this technique is mainly due in the fact that: the motion equations of system are more simplified and the dimensions of the equation are often reduced while the essential behavior of the system is retained.

The primary focus of the current work consists in analyzing the model of Cottone et al. [22] using a probabilistic approach. To that end, we analyze the model equations using a stochastic averaging procedure through Ito's formulation. We also analyze the system using Monte Carlo simulations, thus allowing the relative comparison between numerical and analytical stationary probability density of the output voltage and the escape time that give a view on the stability of the harvester during its functioning. The result of our analysis gives an insight when trying to make predictions about device performance. We first present the basic model of our consideration by giving the description and mathematical modeling of the system. Section 3 presents results from analytic examinations from which we derive the stationary probabilistic response of the system and the numerical scheme used for simulations. In Section 4, we present numerical simulation we have conducted on our systems, followed by a conclusion.

2. Description of the system with the model equation

The mechanical part of energy scavenger of the present investigation showed in Fig. 1 was first studied by Cottone et al. [22]. The kinetic energy of system is defined as:

$$K = \frac{1}{2}\rho_s A_s \int_0^L (\dot{w} + \dot{y})^2 dx + \rho_p A_p \int_0^{L_p} (\dot{w} + \dot{y})^2 dx + \frac{1}{2} M_0 \left(\dot{w}\left(\frac{L}{2}, t\right) + \dot{y}\right)^2 \tag{1}$$

where L and L_p are respectively the length of the substrate and of each piezoelectric layer, $w(x, t)$ is the mid-plan deflection along z axis, ρ_s and ρ_p represents the substrate and piezoelectric materials density, A_s and A_p the cross section of the substrate and piezoelectric materials respectively, M_0 is the mass added at medium of the beam to improve its dynamic vibratory. With the external forces, the electric charges generated by a parallel bimorph is twice the value generated by a series bimorph [22]. However the generated electric voltage in the parallel bimorph is half the value produced by a series bimorph, since the capacitance of the parallel bimorph is four times that of the series bimorph; but in actuation scheme, since the dielectric capacitance in the parallel case is four times that in the series case, power consumption, is the same in both cases [23]. Yabin et al. [24] have also theoretically and experimentally shown that the maximum power output and efficiency is independent of the electrical connection; but can be chosen depending on the value of the load resistance. In this manuscript since considering the series configuration give rise to a coupling scheme that render the analytical probability densities of mechanical deflexion and electrical voltage analytically untractable directly, we adopt the parallel configuration. The piezoceramic layers are assumed to be identical, voltage across the electrodes of each piezoceramic layer is $v(t)$. It is important to add that e_{zx} has the same sign at the top and bottom piezoceramic layers. The instantaneous electric fields are in the opposite directions (i.e., $E_z^u = -\frac{v(t)}{h_p}$ at the top layer and $E_z^l = \frac{v(t)}{h_p}$ at the bottom layer). The total potential energy of system is defined as [22]:

$$E_p = \int_0^L \left(\frac{1}{2} A\left(\epsilon_{xx}^o\right)^2 + B\epsilon_{xx}^o \epsilon_{xx}^1 + \frac{1}{2} D_1\left(\epsilon_{xx}^1\right)^2 - M_p \epsilon_{xx}^1 - b h_p \epsilon^s{}_{zz} E_z^2 - N_p \epsilon_{xx}^o\right) dx - w_e \tag{2}$$

where w_e is the work done by the external compressive force P applied at the movable clamp. The parameters A, B, D, N_p, M_p and w_e are defined as

$$A = 2\, b_p E_p h_p + b_s E_s h_s; \quad B = 0; \quad N_p = 0 \tag{3}$$

$$D_1 = \frac{1}{2}\, b_p E_p h_s^2 h_p + b_p E_p h_s h_p^2 + \frac{2}{3}\, b_p E_p h_p^3 + \frac{1}{12}\, b_s E_s\, h_s^3 \tag{4}$$

$$M_p = -\frac{1}{2} b_p e_{zx}^p \dot{\lambda} h_s - \frac{1}{2}\, b_p e_{zx}^p \dot{\lambda} h_p$$
$$\epsilon_{xx}^o = \frac{1}{2L}\int_0^L w'^2 dx; \; w_e = P\Delta L$$
$$\Delta L = \left(d_b + \frac{1}{2}\int_0^L w'^2 dx\right) \tag{5}$$

d_b represents the contraction length from side pressure corresponding to the critical load P_{cr}. The lagrangian function of the buckled piezoelectric beam is defined by:

$$\pounds = K + T_e - (E_p + T_m) \tag{6}$$