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Pinning adaptive and impulsive synchronization of fractional-order complex dynamical networks

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ABSTRACT

This paper is concerned with the pinning adaptive and impulsive synchronization problem of fractional-order complex dynamical networks. First, a generalized Barbalat's Lemma is derived. Based on the generalized Barbalat's Lemma and some analysis techniques, we obtain some criteria, which guarantee that the whole state-coupled dynamical network can be forced to certain desired synchronous state by combining pinning adaptive control and pinning impulsive control. Finally, numerical simulations are given to demonstrate the effectiveness of the proposed control strategy.

1. Introduction

Complex networks have been widely used to describe various artificial and natural systems, such as the internet networks [1], biological networks [2], neural networks [3], social networks [4], etc. In general, a complex network is composed of a large number of interconnected dynamical nodes, in which each node is a unit with specific contents. In the past few decades, fractional-order complex networks have gained considerable research attention for its more advantage than classical integer-order complex networks in describing the memory and hereditary properties of many materials and processes [5–8]. As an important and interesting collective behavior of complex networks, synchronization has been studied extensively. Note that there are some networks cannot be synchronized by themselves. Then, proper controllers are required for achieving synchronization. So far, many control schemes have been adopted to design effective controllers, such as adaptive control, intermittent control, impulsive control, pinning control, and so on.

It is well known that the advantage of adaptive control is that the control parameters can adjust themselves according to some suitable updating laws, which are designed under control purpose according to the characteristics of considered system [9–11] In [11], Bao et al. investigated the synchronization of fractional-order memristor-based neural networks with time delay by adaptive control. Since the real-world complex networks normally have a large number of nodes, it is usually difficult to control a complex network by adding the controllers to all nodes. To reduce the number of the controllers, a natural approach is to control a complex network by pinning part of nodes [12–14] Recently, by combining the advantages of adaptive control and pinning control, authors of [16–19] investigated synchronization problem of complex networks via adaptive pinning control. In [19], Chai et al. investigated the global synchronization of fractional-order complex networks via adaptive pinning control.

In fact, many practical systems often suddenly receive external disturbance, which makes systems change their trajectories in a very short time. This phenomenon is called impulse. Therefore, the study of the complex dynamical networks with impulsive effects is important for understanding the dynamical behaviors of the most real-world complex networks. There are lots of results about impulsive control for integer-order complex networks, see [20–22] and references therein. Impulsive control of fractional-order complex networks is seldom studied except [23–27]. In [26], Stamova studied the global Mittag–Leffler synchronization of fractional-order neural networks by impulsive control. In [27], Wang et al. investigated the exponential synchronization of fractional-order complex networks by pinning impulsive control. However, to the best of our knowledge, there are no results on the synchronization problem of complex networks via pinning adaptive

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and impulsive control. Motivated by the above discussions, this paper will investigated the pinning adaptive and impulsive synchronization of fractional-order complex dynamical networks.

The main contributions of this paper are the following aspects: (1) We generalize well-known Barbalat's Lemma on the integer-order case to the discontinuous fractional-order case. (2) A new pinning adaptive and impulsive control method is proposed to deal with the synchronization problem of fractional-order complex networks. (3) By using the generalized Barbalat's Lemma and some analysis techniques, sufficient conditions are derived to realize the global synchronization of fractional-order complex networks.

The organization of the paper is as follows. In Section 2, the model formulation and some preliminaries are given. In Section 3, some criteria for the global synchronization of fractional-order complex networks are obtained. In Section 4, a numerical example is provided to illustrate the effectiveness of our theoretical results. In the last Section, we give a brief discussion.

2. Preliminaries and model description

In this paper, let $\mathbb{R}^n$ be the n-dimensional Euclidean space with norm $\|\cdot\|$, and $\mathbb{N}_+ = \{1, 2, 3, \cdots\}$. In this section, some definitions and lemmas are recalled which will be needed later.

Definition 1 [28]**.** The fractional integral of order q for a function f is defined as

$$_{t_0}I_t^q f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} f(s) ds, \quad q > 0,$$

where $\Gamma(\cdot)$ is the well-known Gamma function.

Definition 2 [28]**.** Caputo fractional derivative of order q for a function $f \in C^n([t_0, +\infty), \mathbb{R})$ is defined by

$$_{t_0}^{c}D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{q-n+1}} ds,$$

where $\Gamma(\cdot)$ is the Gamma function, $t \geq t_0$ and n is a positive integer such that $n-1 < q < n$. Particularly, when $0 < q < 1$,

$$_{t_0}^{c}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^{q}} ds.$$

Consider a general complex network consisting of N coupled identical nodes, with each node being a n-dimensional fractional-order dynamical system, which can be described as follows:

$$_{t_0}^{c}D_t^q x_i(t) = f(x_i(t)) + c\sum_{j=1}^{n} a_{ij}\Gamma x_j(t) + u_i, \quad i = 1, 2, \cdots, N, \qquad (1)$$

where $0 < q < 1$, $t_0 \geq 0$ is the initial time, $_{t_0}^{c}D_t^q$ is in the sense of the Caputo fractional derivative, $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node, and $f: \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear vector function describing the nonlinear dynamics of the single node, $u_i \in \mathbb{R}^n$ is controller to be designed later. The positive constant $c > 0$ is the coupling strength, and $\Gamma \in \mathbb{R}^{n\times n}$ is the inner coupling matrix with positive elements. $A = (a_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network, in which a_{ij} is defined as follows: If there is a direct connection from node i to node j, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$, and the diagonal elements of matrix A are defined by $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$.

Let $s(t) \in \mathbb{R}^n$ be a solution of an isolated node system

$$_{t_0}^{c}D_t^q s(t) = f(s(t)). \qquad (2)$$

Our objective is to design some suitable controllers u_i $(i = 1, 2, \cdots, N)$ such that the solutions of the controlled network (1) globally synchronize with the solution of (1), in the sense that

$$\lim_{t\to+\infty} \|x_i(t) - s(t)\| = 0, \quad i = 1, 2, \cdots, N,$$

for any initial conditions.

To derive our main results, we need the following assumption and lemmas.

Assumption 1. There exists a positive constant θ such that for any $x, y \in \mathbb{R}^n$,

$$(x-y)^T(f(x) - f(y)) \leq \theta(x-y)^T(x-y).$$

Lemma 1 [29]**.** *If the Caputo fractional derivative $_{t_0}^{c}D_t^q f(t)$ is integrable, then*

$$_{t_0}I_t^q\, {}_{t_0}^{c}D_t^q f(t) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t_0)}{k!}(t-t_0)^k.$$

Especially, for $0 < q \leq 1$, one can obtain

$$_{t_0}I_t^q\, {}_{t_0}^{c}D_t^q y(t) = y(t) - y(t_0).$$

Lemma 2 [30]**.** *Suppose function $g(t)$ is nondecreasing and differentiable on $t \in [0, \infty)$, then for any constant h and $t \in [0, \infty]$,*

$$_{t_0}^{c}D_t^q \Big(g(t) - h\Big)^2 \leq 2\Big(g(t) - h\Big) {}_{t_0}^{c}D_t^q g(t),$$

where $0 < q < 1$.

Lemma 3 [31]**.** *Let $0 < q \leq 1$. Suppose that $f(t) \in C[a, b]$ and $_{a}^{c}D_t^q f(t) \in C[a, b]$. Then, for all $t \in (a, b]$, there exists $\xi \in (a, t)$ such that*

$$f(t) = f(a) + \frac{1}{\Gamma(q)}\, {}_{a}^{c}D_\xi^q f(\xi)(t-a)^q.$$

Lemma 4 [32]**.** *Let $x(t)$ be a continuous and derivable vector value function. Then for any time instant $t \geq t_0$*

$$\frac{1}{2}\, {}_{t_0}^{c}D_t^q x^T(t)x(t) \leq x^T(t)\, {}_{t_0}^{c}D_t^q x(t), \quad \forall q \in (0, 1),$$

where $0 < q < 1$.

Lemma 5 [33]**.** *Assume that A and B are $N \times N$ Hermitian matrices. Let $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N$, $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N$, $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_N$ be eigenvalues of A, B, and $A+B$, respectively. Then, one has $\alpha_i + \beta_N \leq \gamma_i \leq \alpha_i + \beta_1, i = 1, 2, \cdots, N$.*

Lemma 6 [33]**.** *For a symmetric matrix $M \in \mathbb{R}^{N\times N}$ and a diagonal matrix $D = \mathrm{diag}(d_1, d_2, \cdots, d_l, \underbrace{0, 0, \cdots, 0}_{N-l})$ with $d_i > 0$, $i = 1, 2, \cdots, l$ $(1 \leq l < N)$, let $M - D = \begin{pmatrix} A-\tilde{D} & B \\ B^T & M_l \end{pmatrix}$, where M_l is the minor matrix of M by removing its first l row-column pairs, A and B are matrices with appropriate dimensions, $\tilde{D} = \mathrm{diag}(d_1, d_2, \cdots, d_l)$. If $d_i > \lambda_{\max}(A - BM_l^{-1}B^T)$, $i = 1, 2, \cdots, l$, $M - D < 0$ is equivalent to $M_l < 0$.*

3. Main results

In this section, we introduce a new lemma, which is a generalization of the traditional Barbalat's Lemma [34], and propose an effective control strategy to synchronize the complex network (2) to the desired orbit, which is a solution of system (2).