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Enhancing the precision of measurements in double quantum dot systems via transmission line resonator

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ABSTRACT

Recently, various quantum physical systems have been suggested to control the precision of quantum measurements. Here, we propose a useful quantum system to enhance the precision of the parameter estimation by investigating the problem of estimation in double quantum dot (DQD) spin qubits by considered a transmission line resonator (TLR) as a bus system. To do this, we study the dynamical variation of the quantum Fisher information (QFI) in this scheme including the influence of the different physical parameters. We show that the amount of QFI has a small decay rate in the time and it can be controlled by adjusting the magnetic coupling between DQDs via TLR, initial parameters, and detuning parameter between the qubit system and TLR. These features make DQDs via TLR good candidates for implementation of schemes of quantum computation and coherent information processing.

1. Introduction

One of the most interest objectives of the quantum metrology is to explore the quantum mechanics features for analyzing and examining the parameter-estimation precision, which is recently witnessed much interest and exhibited mandatory requirement in the development of the fields of quantum optics and information. The detection and determination of the rate change of the precision of an unknown parameter initially prepared in a quantum state is the fundamental assignment of the quantum estimation theory (QET) with essential aim is to improve the parameter-estimation parameter. There is a considerable effort in the theoretical and experimental examination of the parameter estimation by deeming several practical quantum aspects such photon loss, decoherence, and practical problems of state generation [1–6]. Fisher information was initially introduced by Fisher [7] that uses a bound to characterize the elements for a probability of distribution and it plays a prominent role in the development of QET. QFI, which characterizes the sensitivity of the state with respect to changes in a parameter, is a key concept in parameter estimation theory. The importance of QET is to provide the optimal measurement for a quantum system that is subjected to an unknown parameter, using the Cramér-Rao inequality (QCR) in which its lower bound is characterized by QFI [8].

Actually, an important goal in solid-state quantum physics is to enhance the amount of the resolution. The motivation behind this quest comes both from the fact that parameter estimation for electrons in a solid-state structure has not yet been proved and from the recent experimental progress in the field of quantum information processing in these systems, leading to experimental realization of single and two-qubit manipulations of electron spin qubits in quantum dots [9–11] and coherent control of spins in diamond [12]. Many aspects of these quantum systems, such as hyperfine coupling to the nuclear spins [13,14] the spin blockade [15,16], realization of the singlet-triplet two-level system by detaining electrons in quantum dot systems placed below the roof of a quantum heterostructure heterostructure [17], and the effects of applying a slanting magnetic field [18] which are currently active topics of research. Double quantum dot systems [19–25] with the phenomenon of TLR [26–32] are particularly attractive because of the relative long spin coherence and high controllability of DQD systems and quantum bus function of the TLR.

Last two decays, several works have been treated the Jaynes Cummings model (JC) with dissipation by the use of analytic approximations and numerical calculations [33–40]. The solution in the presence of dissipation is not only of theoretical interest, but also important from a practical point of view since dissipation would be always present in any experimental realization of the model. However, the dissipation treated in the above studies is modelled by coupling to an external reservoir including energy

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dissipation. As is well known, in a dissipative quantum system, the system loses energy by creating a bath quantum. In this kind of damping the interaction Hamiltonian commutes with that of system and in the dynamics only the phase of system state is changed in the course of interaction. Recently, it has been explored a way to couple electron states in DQD systems in the presence of a TLR with capacitors [41,42]. The interaction Hamiltonian between the qubits and the TLR is a standard JC model [43]. A switchable lagrange interaction can be achieved between any two spatially separated qubits with the TLR. In this paper, we study the dynamical behaviour of the parameter estimation in DQDs using TLR as a bus system in terms of different parameters of the combined system. We focus on the QFI of the DQD with considering the optimal conditions for saturated QCR inequality. We find that the amount QFI can be controlled and has a small decay rate in the time by modifying the magnetic coupling between DQDs via TLR, initial parameters, and detuning between the qubits and the TLR. Such a system can be employed to perform logical operations, which can be used to implement an universal quantum information and computation.

This article is structured as follows. In Section 2, we present a review of the QFI and defining the schematic diagram of the parameter estimation adopter in this paper. In Section 3, we present the model for the DQDs using TLR as a bus system and describe the dependence on different input parameters. Furthermore, we present the related major results with discussion. We conclude our work in Section 4.

2. Quantum estimation theory

In this section, we give a brief summary on the QFI theory. The decisive objective of the QTE is to attain the best observable. When the quantum system will be in one state of the family $\{\rho_\theta\}$, illustrated the true estimated of parameter $\hat{\theta}$, that is $\mathrm{Tr}\left(\rho_\theta\hat{\theta}\right)=\theta$. The QFI measures how precisely a state can detect an unknown parameter and it is given by

$$F_Q=\mathrm{Tr}\left[\rho_\theta L^2\right], \tag{1}$$

where ρ_θ presents the density matrix of the system, θ is the parameter to be measured, and L is the symmetric logarithmic derivation given by

$$\frac{\partial\rho_\theta}{\partial\theta}=\frac{1}{2}[L\rho_\theta+\rho_\theta L], \tag{2}$$

where, the QFI does not depend on the particular choice of L.

The precision bound is asymptotically achieved by the maximum likelihood estimator as well as the classical theory through quantum Cramér–Rao (QCR) inequality,

$$\Delta\theta\geq(\Delta\theta_{\mathrm{QCR}})=\frac{1}{\sqrt{F_Q}}\geq\frac{1}{\sqrt{\langle L^2\rangle}}, \tag{3}$$

where $(\Delta\theta)^2$ is the mean square error in the parameter θ. The above inequality defines the principally smallest possible uncertainty in the value of the parameter estimation. The measurement uncertainty $\Delta\theta$ is quantified through the units corrected, root-mean deviation of the estimate parameter θ from it true value

$$\Delta\theta=\frac{\theta_{\mathrm{est}}}{|d\langle\theta_{\mathrm{est}}\rangle_\theta/d\theta|}-\theta. \tag{4}$$

Consider in general a readout on the probe be described by a POVM with one parameter family of the elements $E(\xi)$

$$\int d\xi E(\xi)=\mathbf{1}. \tag{5}$$

Let $p(\xi|\theta)=\mathrm{tr}(E(\xi)\rho_\theta)$ be the measured probabilities from the various outcomes of the POVM when the true value of the measured parameter is θ. The QFI is given by [44,45]

$$F_Q=\max_{\{E(\xi)\}}F, \tag{6}$$

where F is the classical Fisher information computed from the probability distribution for the measurement outcomes as

$$\begin{aligned}F&=\int d\xi\, p(\xi|\theta)\left[\frac{d\ln p(\xi|\theta)}{d\theta}\right]^2\\&=\int d\xi\,\frac{1}{p(\xi|\theta)}\left[\frac{dp(\xi|\theta)}{d\theta}\right]^2.\end{aligned} \tag{7}$$

The optimization in the Eq. (6) such maximization needs different processes to be done, by performing the POVMs that minimize the measurement uncertainty and lead to maximize the Fisher information. The upper bound on the classical Fisher information is given as [44,45]

$$F\leq\int d\xi\,\mathrm{tr}\left(\frac{E(\xi)\rho_\theta}{\mathrm{tr}(E(\xi)\rho_\theta)}\right).\mathrm{tr}\left(E(\xi)L^2\rho_\theta\right) \tag{8}$$

leading to

$$F_Q=\int d\xi\,\mathrm{tr}\left(E(\xi)L^2\rho_\theta\right)=\mathrm{tr}\left(L^2\rho_\theta\right)=\langle L^2\rangle, \tag{9}$$

and the second inequality in (3) is saturated. This inequality circumvents the maximization problem by placing an upper bound on F_Q in terms of the expectation value of the square of the symmetric logarithmic derivative operator L. This expectation value can be computed directly from the initial state of the probe and its parameter dependent dynamics, independent of the readout procedure.

We choose to compare the precision of the parameter estimation for DQD spin qubits using this widely-accepted approach of QFI. The interferometric set-up generally consists of four steps. The first is the preparation step where the input state is chosen as ρ_{int} for DQD. Then, a singlet-qubit phase gate is applied, given by

$$U_\theta:=|g\rangle\langle g|+e^{i\theta}|e\rangle\langle e|, \tag{10}$$

The output state is obtained by performing a phase gate operator U_θ on the input state; $\rho_{\mathrm{out}}=U_\theta\rho_{\mathrm{int}}U_\theta^\dagger$. After the unitary process, the phase uncertainty is measured for the output mixed state.

The entangled N-qubit states have been proposed as means to beat the so-called shot-noise limit accuracy in parameter estimation [46,47]. The QCR inequality provides a lower limit to the accuracy of estimation $\Delta\theta$ in terms of the inverse of the square of the QFI associated with the generator of the unitary transformation and the state of the system. Now, if ρ_{in} is a separable state, the QFI scales as $O(N)$ with the number of particles in the system, N, while it may scale faster for entangled ρ_{int}.

3. Theoretical model of two DQD spin qubits

The Hamiltonian of the TLR can be written as

$$H_r=\hbar\omega_r a^\dagger a \tag{11}$$

where ω_r is the frequency of the TLR. The Hamiltonian of a DQD with quantization along $z-$axix direction is given by [26,48–50]

$$\begin{aligned}H_D=&E_s|S_{11}\rangle\langle S_{11}|+(\Delta_0+E_S)|S_{02}\rangle\langle S_{02}|\\&+g_B\mu_B B_e\left(|T_{11}^+\rangle\langle T_{11}^+|-|T_{11}^-\rangle\langle T_{11}^-|\right)\\&+E_T|T_{11}^0\rangle\langle T_{11}^0|+t(|S_{11}\rangle\langle S_{02}|+|S_{02}\rangle\langle S_{11}|),\end{aligned} \tag{12}$$

where $\{|S_{11}\rangle,|T_{11}^0\rangle,|T_{11}^+\rangle,|T_{11}^-\rangle,|S_{02}\rangle\}$ presents the two-electron singlet triplet basis are given by

$$|T_{11}^+\rangle=|\uparrow\uparrow\rangle,\quad|T_{11}^0\rangle=\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle),\quad|T_{11}^-\rangle=|\downarrow\downarrow\rangle$$

$$|S_{11}\rangle=\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle),$$