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# Globally asymptotically stable analysis in a discrete time eco-epidemiological system☆

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ABSTRACT

In this study, the dynamical behaviors of a discrete time eco-epidemiological system are discussed. The local stability, bifurcation and chaos are obtained. Moreover, the global asymptotical stability of this system is explored by an iteration scheme. The numerical simulations illustrate the theoretical results and exhibit the complex dynamical behaviors such as flip bifurcation, Hopf bifurcation and chaotic dynamical behaviors. Our main results provide an efficient method to analyze the global asymptotical stability for general three dimensional discrete systems.

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## 1. Introduction

As well known that mathematical models have been widely used to describe the behaviors between different species in natural system [1–6]. Among these models, part of them are used to detect the competitive, cooperation and predator-prey relationships for different species which are called ecological models. Another models are used to explore the outbreak, transmission and extinction behaviors of diseases between the populations which are usually called epidemic models.

However, in some cases, the prey (or predator) species may be infected diseases, and the diseases will spread among the prey and predator species. When the prey specie is infected with the disease the infected prey may be more easily captured by the predator than the susceptible prey, or the predator only eats the infected prey. For example, in Salton Sea of California, the Tilapia fish is infected by a virio class of bacteria, Vibro alginolyticus, which spreads in the fish species and the infected fishes become much easier available for predation for piscivorous birds [7,8]. In this case, the dynamical behaviors between the prey and predator will become more complex because not only the behaviors in the predator-prey process should be considered but also the disease spreading in the prey species must be explored [9]. Therefore, eco-epidemiological systems are needed to address the complex dynamical behaviors between the species.

Recently, more and more attention has been paid on the dynamical behaviors of eco-epidemiological systems [10–22] But most of these works focus on the continuous time systems described by differential equations, few of them are about the discrete time systems described by difference equations. Previous studies [23–42] show that the discrete systems have more complex dynamical behaviors than the corresponding continuous systems except some same behaviors. Moreover, the discrete time systems are more reasonable than the continuous time systems from some perspectives of the natural system [38–42]

In [37], the permanence, global asymptotical stability and Hopf bifurcation of the following continuous time predator-prey system are obtained.

$$\frac{dS}{dt} = rS(1-\frac{S+I}{K}) - \beta SI$$
$$\frac{dI}{dt} = \beta SI - cI - \frac{bIY}{mY+I},$$

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$$\frac{\mathrm{d}Y}{\mathrm{d}t} = -dY + \frac{kbIY}{mY+I}, \tag{1.1}$$

where $S(t)$, $I(t)$ and $Y(t)$ denote the population density of susceptible prey, infected prey and the population density of predator at time $t$, respectively. $r$ is the intrinsic birth rate of the prey population, $K$ is the carrying capacity of the environment about the prey population, $\beta$ is the transmission coefficient, $c$ is the death rate of infected prey, $m$ is the ratio-dependent rate, $b$ is the predation coefficient, $k$ is the coefficient in conversing prey into prey and $d$ is the death rate constant of the predator. The parameters $r$, $K$, $\beta$, $c$, $b$, $m$, $d$ are positive constant and $k$ is satisfied $k \in (0, 1]$.

In our previous study [9], we discussed the local stability of a discrete time system discretized from system (1.1). If the bilinear incidence rate $\beta SI$ becomes into the standard incidence rate $\frac{\beta SI}{S+I}$ then how to analyze the dynamical behaviors. Therefore, in this paper, the following discrete time system with standard incidence rate $\frac{\beta SI}{S+I}$ is considered.

$$\begin{aligned}
S_{n+1} &= S_n \exp\left[r\left(1-\frac{S_n+I_n}{K}\right) - \frac{\beta I_n}{S_n+I_n}\right],\\
I_{n+1} &= I_n \exp\left[\frac{\beta S_n}{S_n+I_n} - c - \frac{bY_n}{mY_n+I_n}\right],\\
Y_{n+1} &= Y_n \exp\left[\frac{kbI_n}{mY_n+I_n} - d\right],
\end{aligned} \tag{1.2}$$

where $r$, $K$, $\beta$, $c$, $b$, $m$, $d$ and $k$ are defined as in system (1.1). It is assumed that the initial value of system (1.2) $S_0 > 0$, $I_0 > 0$, $Y_0 > 0$ and all the parameters are positive. Obviously, if the initial value $(S_0, I_0, Y_0)$ is positive, then the corresponding solution $(S_n, I_n, Y_n)$ is positive too.

It is known that the global asymptotical stability is an important dynamical behavior for species systems. However, for a three dimensional discrete system (as system (1.2) in this study) it is difficult to prove it. Our previous study [9] only discussed the locally stability. Therefore, in this study, we will address the following questions: (1) How to analyze the local stability of system (1.2)? (2) Which parameters mainly control the dynamical behaviors variations, such as flip bifurcation, Hopf bifurcation and chaos. (3) Is there an efficient method to obtain the global asymptotical stability of system (1.2), and whether this method can be extended to three dimensional discrete system?

The organization of this study is as follows. In the second section the existence and local stabilities of equilibria in system (1.2) are discussed. In the third section, the global asymptotical stability of equilibria will be proved by an iteration scheme. In the fourth section we present the numerical simulations, which not only illustrate our results with the theoretical analysis, but also exhibit the complex dynamical behaviors. The conclusion is provided in the last section.

## 2. Analysis of equilibria

For system (1.2) we always assume that $S_0 > 0$, $I_0 > 0$, $Y_0 > 0$ and all the parameters are positive, then it is obviously any solutions of system (1.2) are positive for all $n \geq 0$.

Firstly, on the existence of the nonnegative equilibria of system (1.2), we have the following results.

**Theorem 1.**

(1) *System (1.2) always has two equilibria $E_0(0, 0, 0)$ and $E_1(K, 0, 0)$.*
(2) *When $c < \beta < c + r$ and $bk \leq d$, $E_2(\frac{cK(r+c-\beta)}{r\beta}, \frac{K(\beta-c)(r+c-\beta)}{r\beta}, 0)$ is another equilibrium of system (1.2).*
(3) *When $bk > d$ and $f < \beta < r + c$, besides $E_0$, $E_1$ and $E_2$, system (1.2) has one positive equilibrium $E_3(S^*, I^*, Y^*)$, where*

$$S^* = \frac{fI^*}{\beta-f}, \quad I^* = \frac{K(\beta-f)(r+f-\beta)}{r\beta},$$
$$Y^* = \frac{bk-d}{md}I^*, \quad f = c + \frac{bk-d}{mk}.$$

Theorem 1 shows that the equilibria can be simultaneous existence for certain parameter ranges, such as the results (2) and (3). However, they have different stabilities with the variations of the parameters. When the parameters change, some equilibria may be stable and the others will become unstable. These will be obtained according to their stable analyses.

Now, we study the stabilities of equilibria $E_0$, $E_1$, $E_2$ and $E_3(S^*, I^*, Y^*)$ of system (1.2). The stabilities of $E_0(0, 0, 0)$ and $E_1(K, 0, 0)$ can be obtained by a similar discussion as in [9]. Therefore, in this study we only focus on the dynamical behaviors of $E_2$ and $E_3(S^*, I^*, Y^*)$, including local stability, global asymptotical stability and complex dynamical behaviors (bifurcation and chaos).

The Jacobian matrix of system (1.2) at an equilibrium $E(S, I, Y)$ is $J(E) = J(E_j)$ where $j = 0, 1, 2, 3$. Let $w_1$, $w_2$ amd $w_3$ are the three eigenvalues of matrix $J(E_j)$, we have the following definitions.

(1) If $|w_1| < 1$, $|w_2| < 1$ and $|w_3| < 1$, then $E(S, I, Y)$ is called a sink and is locally asymptotically stable;
(2) If $|w_1| > 1$, $|w_2| > 1$ and $|w_3| > 1$, then $E(S, I, Y)$ is called a source and is unstable;
(3) If $|w_1| > 1$, $|w_2| > 1$ and $|w_3| < 1$ (or $|w_1| < 1$, $|w_2| > 1$ and $|w_3| > 1$), then $E(S, I, Y)$ is called a saddle and is unstable;
(4) If $|w_1| = 1$ or $|w_2| = 1$ or $|w_3| = 1$, then $E(S, I, Y)$ is called non-hyperbolic.

For equilibrium $E_2(\frac{cK(c+r-\beta)}{r\beta}, \frac{K(\beta-c)(c+r-\beta)}{r\beta}, 0)$, the characteristic equation of $J(E_2)$ can be computed as

$$f(w) = (w - e^{(kb-d)})(w^2 + Bw + C),$$

where

$$B = -\left(2 - \frac{rS}{K}\right), \quad C = 1 - \frac{rS}{K} + \frac{r\beta SI}{K(S+I)}.$$

Obviously, according to $bk \leq d$, $f(w)$ has one eigenvalue $w_1 = e^{(kb-d)}$ with $0 < w_1 \leq 1$. Further, we note $g(w) = w^2 + Bw + C$. For detecting the stability of equilibrium $E_2(\frac{cK(c+r-\beta)}{r\beta}, \frac{K(\beta-c)(c+r-\beta)}{r\beta}, 0)$, the roots of $g(w) = 0$ need be detected. Supposing the roots of $g(w) = 0$ are $w_{2,3}$. We use the following result which can be easily proved by the relations between roots and coefficients of the quadratic equation.

**Lemma 1.** *(see [43]) Let $F(w) = w^2 + Bw + C$, where $B$ and $C$ are constants. Supposing $F(1) > 0$ and $w_1$, $w_2$ are two roots of $F(w) = 0$. Then*

(1) *$|w_1| < 1$ and $|w_2| < 1$ if and only if $F(-1) > 0$ and $C < 1$;*
(2) *$w_1 = -1$ and $|w_2| \neq 1$ if and only if $F(-1) = 0$, $B \neq 0, 2$;*
(3) *$|w_1| < 1$ and $|w_2| > 1$ if and only if $F(-1) < 0$;*
(4) *$|w_1| > 1$ and $|w_2| > 1$ if and only if $F(-1) > 0$ and $C > 1$;*
(5) *$w_1$ and $w_2$ are the conjugate complex roots and $|w_1| = |w_2| = 1$ if and only if $B^2 - 4C < 0$ and $C = 1$.*

Now, by computing, we obtain $g(1) = \frac{r\beta SI}{K(S+I)} > 0$. Further,

$$g(-1) = 4 - \frac{2rS}{K} + \frac{r\beta SI}{K(S+I)}.$$

From $g(-1) = 0$, we have $r = r^*$, $\beta = \beta_1, \beta_2$, where

$$r^* = \frac{4\beta - c(c-\beta)(2+c-\beta)}{c(2+c-\beta)},$$

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