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A semi-analytical iterative method for solving nonlinear thin film flow problems

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ABSTRACT

This paper presents a new implementation of a reliable iterative method proposed by Temimi and Ansari namely (TAM) for approximate solutions of a nonlinear problem that arises in the thin film flow of a third grade fluid on a moving belt. The solution is obtained in the form of a series that converges to the exact solution with easily computed components, without any restrictive assumptions for nonlinear terms. The results are bench-marked against a numerical solution based on the classical Runge–Kutta method (RK4) and an excellent agreement is observed. Error analysis of the approximate solution is performed using the error remainder and the maximal error remainder. An exponential rate for the convergence is achieved. A symbolic manipulator Mathematica ®10 was used to evaluate terms in the iterative process.

1. Introduction

Nonlinear differential equations play an important role in modeling numerous important phenomena occurring in various fields of physics, chemistry and engineering science, and are frequently modeled through nonlinear differential equations.

Various methods have been used to solve linear and nonlinear differential equations such as Adomain decomposition method (ADM), variational iteration method (VIM), homotopy perturbation method (HPM), homotopy analysis method (HAM) and some other analytical and approximate methods.

Many problems in fluid mechanics are modeled by nonlinear differential equations and the exact solution is difficult or impossible to obtain, therefore, approximate and numerical methods are used to handle these type of problems [1].

The flow and heat transfer phenomena of a viscous liquid over a stretching surface have promising applications in a number of technological processes such as metal and polymer extrusion, continuous casting and drawing of plastic sheets [2,3].

Also, Wang and Pop [4] have studied the flow of a power-law fluid film on an unsteady stretching surface by HAM. Liu and Anderson [5] have examined the heat transfer in a liquid film driven by a horizontal sheet.

The fluids classically known as Newtonian, at constant temperature and pressure, and in simple shear, the shear stress is proportional to the rate of shear and the constant of proportionality is the familiar dynamic viscosity. Newtonian fluid means there is a linear relationship between shear stress and the rate of shear, otherwise, the fluid is non-Newtonian when the relation is nonlinear or complex [6].

The solution of Newtonian and non-Newtonian fluids have been rapidly increased due to the importance of these kind of fluids in practical engineering problems [7–10]. Many attempts have been made to develop analytical and approximate methods to solve nonlinear thin film flow problems, such as ADM and VIM [2,6], HPM [3,11], HAM [12].

Recently, Temimi and Ansari [13] have introduced a semi-analytical iterative technique namely (TAM) for solving nonlinear problems. The TAM is used for solving many differential equations, such as nonlinear second order multi-point boundary value problems [14], nonlinear ordinary differential equations [15]. AL-Jawary et al. have successfully applied the TAM for Duffing equations [16] and some chemistry problems [17], and the results obtained from the method indicate that the TAM is accurate, fast, appropriate, and has a higher convergence.

In this paper, the TAM will be applied to solve nonlinear thin film flow problems. Special discussion is given for the study of convergence based on [15] and the error analysis of the TAM.

This paper has been organized as follows: In Section 2, the nonlinear thin film flow problems (NTFFPs) will be introduced. In Section 3, the basic idea of TAM is presented and discussed. In Section 4, solving the NTFFPs by the TAM will be given. In Section 5, the convergence and error analysis are introduced and discussed. In Section 6, the numerical simulation will be illustrated and discussed. Finally, the conclusion is given in Section 7.

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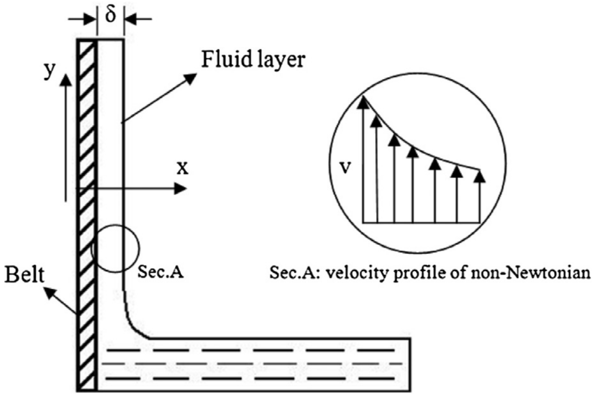

Fig. 1. Physical sketch of the flow of moving belt through a non-Newtonian fluid [6].

2. Nonlinear thin film flow problems

The following nonlinear boundary value problem presents the thin film flow of a third grade fluid on a moving belt [1,11]:

$$\frac{d^2v}{dx^2} + \frac{6(k_2+k_3)}{\mu}\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2} - \frac{\rho g}{\mu} = 0, \tag{1}$$

$$v(0) = k,\ \frac{dv}{dx} = 0,\ \text{at } x = \delta, \tag{2}$$

where v is the fluid velocity which is a function of x only, ρ the density, μ the dynamic viscosity, k_2 and k_3 are material constants of the third grade fluid, g the acceleration due to gravity, δ the uniform thickness of the fluid film and k the speed of the belt as shown in Fig. (1). For simplicity, some assumptions are made: the flow is in steady state, the flow is laminar and uniform, and the film fluid thickness δ is uniform [6].

The following dimensionless variables will be introduced

$$\hat{x} = \frac{x}{\delta},\ \hat{v} = \frac{v}{k},\ \beta = \frac{(k_2+k_3)k^2}{\mu\delta^2},\ m = \frac{\rho g\delta^2}{\mu k}.$$

Then, Eqs. (1) and (2) will be reduced to the following system, ignoring the symbol ˆ,

$$\frac{d^2v}{dx^2} + 6\beta\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2} - m = 0, \tag{3}$$

with the boundary conditions

$$v(0) = 1,\ \frac{dv}{dx} = 0,\ \text{at } x = 1, \tag{4}$$

Eq. (3) is a well-posed second order nonlinear ODE. By integrating Eq. (3) once with respect to x, we get:

$$\frac{dv}{dx} + 2\beta\left(\frac{dv}{dx}\right)^3 - mx = c_1, \tag{5}$$

and applying the second boundary condition in Eq. (4), we obtain $c_1 = -m$, therefore, the following first order nonlinear ODE is obtained:

$$\frac{dv}{dx} + 2\beta\left(\frac{dv}{dx}\right)^3 - m(x-1) = 0, \tag{6}$$

$$v(0) = 1, \tag{7}$$

It is worth to mention that, when $\beta = 0$, Eq. (1) reduces to the Newtonian fluid case [1,11]. Also, for simplicity, accuracy and reducing the computational time, we will solve Eqs. (6) and (7) by the TAM rather than Eqs. (3) and (4).

3. The basic idea of TAM

Temimi and Ansari [13–15] have introduced the semi-analytical method TAM which can be summarized by the following:

Let us consider the general differential equation

$$L(v(x)) + N(v(x)) + g(x) = 0, \tag{8}$$

with boundary condition

$$B(v, \frac{dv}{dx}) = 0 \tag{9}$$

where x indicates the independent variable, $v(x)$ is an unknown function, $g(x)$ is a known function, L is a linear operator, N is a nonlinear operator. Here, we can take linear parts and add them to N as needed. The method is applied as follows: we start by assuming that $v_0(x)$ is an initial approximation of the solution $v(x)$ to the equation, and is the solution of the linearized equation.

$$L(v_0(x)) + g(x) = 0, \quad \text{with} \quad B\left(v_0, \frac{dv_0}{dx}\right) = 0 \tag{10}$$

To find the next iterate to the solution, we solve the following equation:

$$L(v_1(x)) + g(x) + N(v_0(x)) = 0, \quad \text{with} \quad B\left(v_1, \frac{dv_1}{dx}\right) = 0 \tag{11}$$

Thus, we have a simple iterative step which is improving the solution of a linear set of equations,

$$L(v_{n+1}(x)) + g(x) + N(v_n(x)) = 0, \quad \text{with} \quad B\left(v_{n+1}, \frac{dv_{n+1}}{dx}\right) = 0 \tag{12}$$

It is noted that each of the $v_i(x)$ are solutions to Eq. (8). Thus, evaluating more approximate terms, provides better accuracy.

4. Convergence and error analysis

In order to discuss the convergence and error analysis for TAM applied to NTFFPs, we follow a similar procedure as given for a second-order nonlinear ODE with some modifications [15]. Define the following L^2 norm

$$\|f\| = \left(\int_0^t f^2\right)^{\frac{1}{2}} \tag{13}$$

The error remainder is given by [16]

$$ER_n = \frac{d^2v_n}{dx^2} + 6\beta\left(\frac{dv_n}{dx}\right)^2\frac{d^2v}{dx^2} - m \tag{14}$$

The maximal error remainder parameters are:

$$MER_n = \max_{0\le x\le 1}!`|ER_n(x)|, \tag{15}$$

Consider the thin film flow problem

$$\frac{d^2v}{dx^2} + 6\beta\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2} - m = 0 \tag{16}$$

with the following boundary condition

$$v(0) = 1,\ \frac{dv}{dx} = 0,\ \text{at } x = 1, \tag{17}$$

The main aim is to prove that the sequence of functions v_n, which are solutions of [16]

$$\frac{d^2v_{n+1}}{dx^2} + 6\beta\left(\frac{dv_n}{dx}\right)^2\frac{d^2v_n}{dx^2} - m = 0 \tag{18}$$