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Letter to the editor

Complex dynamics of a bistable electrically charged microcantilever: Transition from single well to cross well oscillations

1. Introduction

Energy harvesting from ambient mechanical vibrations is important for remote electronic devices to fully achieve their energy autonomy [1–4]. Cordless and wireless sensor systems are desirable, and this can only be accomplished by using batteries and/or harvested energy. Vibration energy harvesting (VEH) is a useful technology for those modern electronic applications, as the energy captured from the ambient can be used directly or used recycled to, at least, recharge batteries or other storage devices, which enhances battery life and, consequently, to extend the device autonomy in terms of time between recharges [1]. Earlier results were obtained by using cantilever beams with piezoelectric, electrostatic, or electromagnetic coupling [5]. In a simplified case, the mono frequency excitation provided by narrow-band energy sources is engaged leading to resonance-based energy harvesting [5,6]. Therefore, the resonance frequency of the energy harvesting device must be tuned to the characteristic frequency of the available ambient energy source since, in most of the cases, this frequency is not well determined or it can be time varying. In order to implement such a requirement, several self-tuning strategies have been developed up to now. Some of them consist on passive solutions mechanically achieved through special geometries of the transducer harvester element [7]. Some others imply additional active circuitry and are based on feedback functionalities added to the power management electronic part [8]. Alternatively, a number of non-linear devices have been proposed to maximise harvested energy over a wide range of excitation frequencies [10], even in the case of random vibration sources characterized by a wideband spectral density [9]. On the other hand, the intensive developments are directed to miniaturization [11]. Using electrical and magnetic interactions, the researchers were able to scale down the cantilever beams with additional charges (electrical or magnetic dipoles) [12,13]. Simultaneously, recent material developments indicate that structured nanowires collected in the arrays could be promising [14,15].

Among them an important group is based on bistable configurations [12,16–19]. For a large enough excitation amplitude they obtained a cross barrier dynamics and, consequently, a large electrical power output. Furthermore, the device can work efficiently in fairly lower frequency ambient vibration conditions [17,19]. In the present paper we continue this direction of studies using a small scale approach.

Our model is based on a recent proposal [12], which is characterized by an additional horizontal shift a of the cantilever (Fig. 1) with respect to the frame. This shift brakes the left-right reflecting symmetry of the potential.

After defining the displacement of the cantilever free end by x (Fig. 1), the total potential is following (Fig. 2a):

$$V = k_s x^2/2 + Kq^2/\sqrt{(x-a)^2+d^2}. \quad (1)$$

The system parameters where taken from [12] and a= 0.6 μm and d=3 μm. The stable (x_1, x_2) and unstable (x_0) equilibrium points are defined for the corresponding potential mimima and maximum:

$$\frac{dV(x)}{dx} = 0 \quad (2)$$

$$x_1 = -2.7 \times 10^{-6}\ \text{m},\ x_2 = 3.0 \times 10^{-6}\ \text{m} \quad \text{and } x_0 = 9.7 \times 10^{-7}\ \text{m}. \quad (3)$$

The equations of motion can be written:

$$\dot{x} = v, \qquad m\dot{v} = -\frac{dV}{dx} - \beta v + f_0 cos(\omega t), \quad (4)$$

where m, β, f_0 and ω are the reduced modal mas, damping coefficient, amplitude and frequency of excitation.

2. Homoclinic orbits and Melnikov criterion

Considering small perturbations (ϵ-small parameter) to the hamiltonian system the equations read

$$\dot{x} = v, \qquad m\dot{v} = -\frac{dV}{dx} - \epsilon\beta v + \epsilon f_0 \cos(\omega t), \quad (5)$$

For $\epsilon = 0$ one can obtain homoclinic orbits - solutions stating and finishing at the saddle point for time $t \to \pm\infty$ as the homoclinic orbit $[x^*(t), v^*(t)]$ ploted in Fig. 2b.

To examine the escape phenomenon we define the Melnikov [20–22] function as:

$$M(t_0) = \left| \int_{-\infty}^{+\infty} v^*(t)(f_0 \cos(\omega t) - \beta v^*(t))dt \right|. \quad (6)$$

where t_0 is the time-like parameter to be chosen to minimize the function. Note that $M(t_0)$ is proportional to perturbed stable and unstable manifolds with respect to the saddle point x_0 (Eq. (3)). Crossing of such stable and unstable manifold implies a homoclinc bifurcation. $M(t_0)$ has a meaning of the distance d between stable and unstable manifolds (see Fig. 3).

Consequently, a condition for a global homoclinic transition, corresponding to a horseshoe type, can be written as [20]:

$$\bigvee_{t_0} M(t_0) = 0 \quad \text{and} \quad \frac{\partial M(t_0)}{\partial t_0} \neq 0. \quad (7)$$

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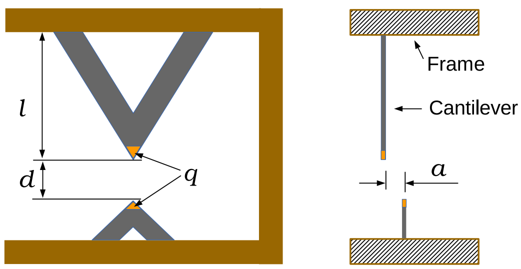

Fig. 1. Scheme of the experimental system after Ref. [12]. A V-shaped cantilever (top) (l = 200 μm; d from 2.5 μm to 10 μm) and a counter electrode (bottom) are permanent and locally charged with q, at their free ends and separated by a distance $\sqrt{d^2+a^2}$. In the studied case for the charge and reduced modal mass were used $q = 1.1 \times 10^{-14}$C and $m = 1.0 \times 10^{-11}$ kg, respectively.

Table 1
Resonator signal output in terms of corresponding variance.

Case	Variance of displacement (μm)^2	Variance of velocity (mm/s)^2
(a)	8.94	1.70
(b)	1.75	0.18
(c)	1.51	1.34
(d)	9.65	1.70
(e)	9.02	2.00
(f)	7.44	2.00

The border curve distinguishing the single well and cross well oscillation regions for two homoclinic orbits are present in Fig. 2c. The region of system parameters for single well solutions [22] are below both curves (blue and red) denoting the escape criterion for left and right hand side potential wells (Fig. 2a), respectively.

3. Simulation results: phase portraits and corresponding time series

One of the standard methods of identification of nonlinear dynamics is the phase portrait and Poincare map [23]. Fig. 4 shows the portraits of six selected simulations corresponding to different choices of excitation amplitude and frequency. The corresponding time series of displacement are shown in Fig. 5.

By changing the amplitude (see Fig. 4b and e or 4 c and f) for a given frequency of excitation, we observe the transition from single well to cross well oscillations. In case of low frequency (see Fig. 4a and b) the oscillations are already extended to both potential wells. Interestingly, Fig. 4f implies chaotic vibrations. This is clear from the observation of phase trajectories which are closed in Fig. 4a–e and open in Fig. 4f. Additional indication can be made from the Poincare (stroboscopic) points which are singular and limited in Fig. 4a–e, while from Fig. 4f the distribution is extended into a large region of phase space. Displacement and variance outputs of this micro-mechanical resonator are presented in Table 1. Note that the displacement indicate more significant difference caused by transition from single to cross-well oscillations (see cases b and e or c and f). This could imply the possible application to energy harvesting by using displacement sensitive transducer as piezoelectric or electrostatic one. The displacement time series (Fig. 5) of the corresponding cases confirm the above observations.

Finally, the frequency spectra are presented in Fig. 6a–f. As expected, the forcing frequency and corresponding higher harmonics are clearly visible as high peaks in all the cases. This is due to nonlinearities in the system. Note that the Fourier spectra are composed of peaks and also filled space between them for a non-periodic case (Fig. 6f) or have just discrete peak structures for the periodic cases (see Fig. 6a–e).

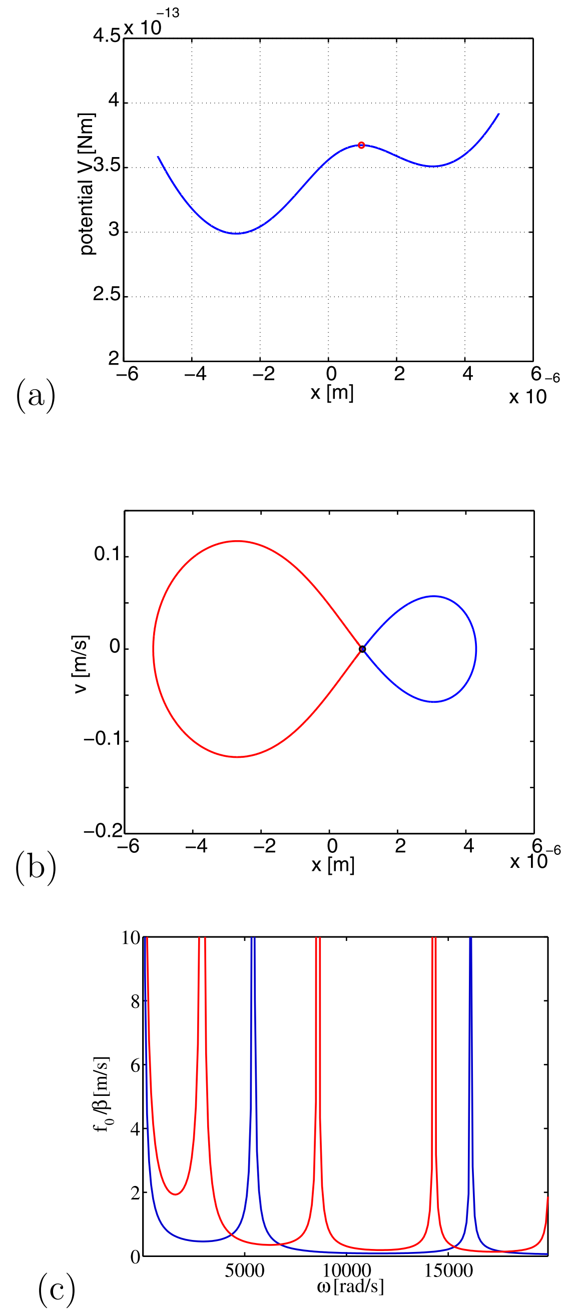

Fig. 2. (color online) Potential shape with the saddle point (a), homoclinic orbits (b), results from Melnikov criterion (c) – above the critical curve there is an escape from the potential well, blue and red colours denote escape criterion for left and right potential wells, respectively. x and v defines the displacement and velocity of the cantilever free end (Fig. 1), ω is the excitation frequency. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Multi scale entropy analyses

To improve understanding of the behaviour of complex systems that manifest themselves, sample entropy analysis becomes increasingly popular [19]. This method provides, for measured signals, a relative level of complexity of finite length time series. The concept of Multi-Scale Entropy (MSE) is based on the coarse-graining procedure that uses a coarse-grained time series, as an average of the original data points within non-overlapping windows by increasing the scale factor τ according to the