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On the generation of spiral and scroll waves by periodic stimulation of excitable media in the presence of obstacles of minimum size

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ABSTRACT

In this work we consider the periodic stimulation of two and three dimensional excitable media in the presence of obstacles with an emphasis on cardiac dynamics. We focus our attention in the understanding of the minimum size obstacles that allow generation of spiral and scroll waves, and describe different mechanisms that lead to the formation of such waves. The present study might be helpful in understanding and controlling the appearance of spiral and scroll waves in the medium.

1. Introduction

The propagation of waves in excitable media is a very important phenomenon to study due to its relation to health problems such as cardiac arrhythmias [1,2], gradual loss of visual acuity [3], seizures in neocortex [4] as well as in understanding the behavior of aggregating Dictyostelium discoideum amoebae [5], giant honeybee colonies [6], chemical reactions [7] and in the retina [8]. Under proper initial conditions, self sustained rotating waves, known as spiral and scroll waves in two and three dimensions, respectively, are obtained. A particular problem arises in cardiac physiology when scroll waves become unstable leading to a possible genesis for ventricular fibrillation [2].

Spiral waves have been obtained in computational models by stimulating periodically a medium which has non excitable obstacles [9], by a cut wave front, cross-stimulation or the phase distribution method [10]. Spatially coherent spiral waves can emerge out of uncorrelated noisy disturbances [11] and from the application of subthreshold perturbations with internal stochasticity [12]; in this case, the waves are remarkably robust if subject to periodic forcing [13]. In chemistry, they have been obtained in the BZ reaction by breaking up target waves [14]. For the same reaction, scroll waves have been created in a two layer preparation where after expanding a spherical wave induced in the interface, and mixing the top part obtaining a homogeneous medium, the birth of a scroll ring was induced [7]. Finally, in cardiac physiology spiral waves might arise due to wavefront-obstacle interactions [15], due to electromechanoelectrical feedback [16], the appearance of ectopic beats [17] which are originated due to abnormal calcium cycling [18], by overload of calcium inside the cell [19] or by the inhibition of the rectifier potassium current [20]. In general, methods of generation of scroll waves are inherited from the mechanisms for spiral waves.

In order to control or eliminate such waves, different methods have been developed [10,21–25]. However, an important endeavor is the understanding of particular mechanisms of the origin of spiral and scroll waves, so it might be possible to avoid their generation instead of controlling their behavior. Particularly, the generation of scroll waves by periodic stimulation of a medium in the presence of obstacles is addressed in this work. The relevance of generation of spiral waves due to obstacles can be appreciated from cardiac dynamics, where obstacles might appear from scar tissue, product of previous infarctions [15,26], or can be arteries [27] or natural orifices in the atria [28]. In this case, obstacles play an important role in the transition and evolution of different cardiac arrhythmias [29].

In a seminal work [9], Panfilov and Keener presented a study about the generation of spiral waves, when periodic propagating pulses interacted with a non excitable obstacle. When an excitable medium with an obstacle was excited periodically, it was shown that for a certain periodicity in the stimulation impulses, a pair of symmetric spiral waves were created [9]. Moreover, one of the

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conclusions was that a minimum length of the obstacle in the direction perpendicular to the course of the stimulation pulses, is required in order to obtain spiral waves.

Although the mechanism they presented is well known in the literature, there are still questions that have not been addressed properly. For instance, is the proposed mechanisms in [9] the only one that implies generation of spiral waves? In which sense are the results in [9] still valid in the three dimensional space? In this work we are interested to find properties in the size of rectangular and rectangular prism obstacles in two and three dimensions, respectively, such that under periodic stimulation we obtain, or not, spiral and scroll waves. The mechanisms of generation of spiral and scroll waves will depend on the size of the obstacle, the excitability of the medium and the dimension of the space. The results in the three dimensional space are not a straightforward extension of the two dimensional case.

Thus, this work is organized as follows. Initially, in Section 2 we present the model equations and the numerical methods used in this work. After that, in Section 3, an analysis of the minimum size that an obstacle in a two dimensional space requires in order to form spiral waves as a function of the excitability of the medium, is presented. In the same section, we extend the studies to the three dimensional space where two different types of generated scrolls (attached and meandering) are presented and discussed. Additionally, two different mechanisms to obtain meandering scroll waves are discussed. We close this work with a discussion and conclusions section (Section 4).

2. The model equations and numerical methods

For the numerical simulations we used the model presented by Panfilov and Keener [9], which is of the FHN type [30,31] and it was developed to understand cardiac dynamics. We employed these equations as our motivation are cardiac dynamics and our results will be argued using generic properties of excitable media. However, despite their simplicity it has been shown that some of the results obtained with these simple models can be used to understand properties for more complex models of excitable nature. The equations are given by

$$\begin{aligned} \dot{u} &= \nabla \cdot (D\nabla u) - f(u) - v \\ \dot{v} &= \varepsilon(u)(ku - v) \end{aligned} \tag{1}$$

where we take $D = I$, the identity matrix in two or three dimensions, and

$$f(u) = \begin{cases} C_1 u & \text{for} \quad u \le u_1 \\ -C_2 u + a & \text{for} \quad u_1 < u \le u_2 \\ C_3(u-1) & \text{for} \quad u > u_2 \end{cases}$$

$$\varepsilon(u) = \begin{cases} \varepsilon_1 & \text{for} \quad u \le u_1 \\ \varepsilon_2 & \text{for} \quad u_1 < u \le u_2 \\ \varepsilon_3 & \text{for} \quad u > u_2 \end{cases}.$$

The constants C_1, C_2, C_3, a, k, ε_1, ε_2 y ε_3 are positive and $u_1, u_2 \in (0, 1)$. In our simulations the parameter values were taken as $u_2 = 0.841$, $C_1 = 20$, $C_2 = 3$, $C_3 = 15$, $a = 0.15$, $\varepsilon_1 = 0.14$, $\varepsilon_2 = 0.0589$, $\varepsilon_3 = 2.5$, and u_1 varied. u_1 is one of the parameters that controls the threshold value for excitation. A larger value of u_1 implies a less excitable system. The variable u is the excitable variable and v is the inhibitory variable. In cardiac dynamics u plays the role of the membrane potential, whereas v represents a gate variable.

The set of Eq. (1) are solved in two and three dimensions. In two dimensions the traditional centered finite difference was used to discretize the second derivative in space, whereas advance in time was done with the Euler method. In some of the three dimensional simulations (Fig. 6), a semi-implicit scheme was implemented as given by Keener and Bogar [32], in order to accelerate the computations. For the explicit finite differences scheme,

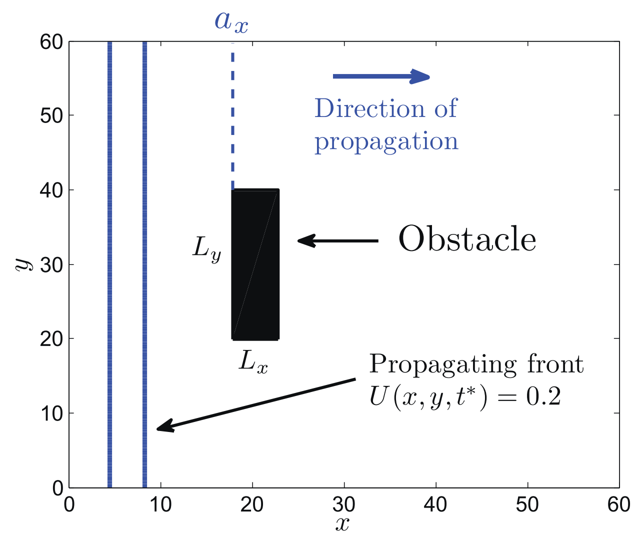

Fig. 1. Obstacle in the two dimensional space. Propagation of the pulses takes place from left to right. By moving a_x, the length of the obstacle in the x direction changes.

the time step is taken as $\Delta t = 0.001$ and the grid discretization is $\Delta x = 0.1$, whereas for the semi-implicit scheme $\Delta t = 0.02$ and $\Delta x = 0.13$. From [9], we have that 1 space unit corresponds to 2.4 mm and 1 computational time unit equals 10.59 ms.

Finally, obstacles in excitable media with emphasis of cardiac wave propagation, can have a partially excitable or non-excitable nature [29] and in this work we focus our study in the latter ones. For the solution of the equations, no-flux boundary conditions are imposed at the boundary of the obstacle and the domain.

3. Numerical results

3.1. Two dimensional studies

In this section we present a study of the minimum size of an obstacle required to form a spiral wave as a function of u_1. We consider initially a rectangular domain $\Omega = \{(x, y)|x \in [0, 60], y \in [0, 60]\}$ and an obstacle inside the domain given by $\Omega^o = \{(x, y)|x \in [a_x, 23], y \in [30 - L_y/2, 30 + L_y/2]\}$, where a_x is a parameter value that will help to choose the length of the obstacle in the x direction, by moving the left part of the obstacle and keeping fixed the right side of the obstacle at $x = 23$ (Fig. 1).

3.1.1. Excitability threshold

One of the measures of excitability is given by the threshold potential [31]. The threshold potential plays an essential role in the generation and propagation of action potentials and it is a measure between the resting potential and the minimum potential needed to generate an action potential. In a medium with a reduced threshold potential, new action potentials can be generated and propagate more easily than in the case of an increased threshold potential. As the threshold potential is increased, it becomes more difficult to activate the medium to generate new action potentials.

In the model, u_1 provides a measure of the threshold potential, and the effect of modifying it can be observed in the trajectory traced by the tip of a spiral wave. Fig. 2 shows different tip trajectories for particular values of u_1 in the interval [0.0065, 0.136]. Values of u_1 larger than 0.136 result in circular tip trajectories with increasing radii, and for $u_1 > 0.152$ no spiral waves can be formed. Tip trajectories are computed by the intersection of