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Synchronization of bursting neurons with a slowly varying d. c. current

Ranjit Kumar Upadhyay*, Argha Mondal

Department of Applied Mathematics, Indian Institute of Technology (Indian School of Mines), Dhanbad-826004, India

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ABSTRACT

Bursting of neuronal firing is an interesting dynamical consequences depending on fast/slow dynamics. Certain cells in different brain regions produce spike-burst activity. We study such firing activity and its transitions to synchronization using identical as well as non-identical coupled bursting Morris-Lecar (M-L) neurons. Synchronization of different firing activity is a multi-time-scale phenomenon and burst synchronization presents the precursor to spike synchronization. Chemical synapses are one of the dynamical means of information processing between neurons. Electrical synapses play a major role for synchronous activity in a certain network of neurons. Synaptically coupled neural cells exhibit different types of synchronization such as in phase or anti-phase depending on the nature and strength of coupling functions and the synchronization regimes are analyzed by similarity functions. The sequential transitions to synchronization regime are examined by the maximum transverse Lyapunov exponents. Synchronization of voltage traces of two types of planar bursting mechanisms is explored for both kind of synapses under realistic conditions. The noisy influence effects on the transmission of signals and strongly acts to the firing activity (such as periodic firing and bursting) and integration of signals for a network. It has been examined using the mean interspike interval analysis. The transition to synchronization states of coupled and a network of bursting neurons may be useful for further research in information processing and even the origins of certain neurological disorders.

1. Introduction

Physical systems have close frequencies interaction. In contrast to this, many biological systems are composed of a fast and slow systems [1,2]. The geometric theory of singular perturbations [3,4] is a powerful tool for analyzing such systems with three or more dynamical variables near or away from the equilibrium point. The study of the organization of vector field near the equilibrium point is known as the local version of the theory. On the contrary, behavior of the system away from the equilibrium point is the global version of the theory. Invariant manifolds provide us an idea about how separatrices would organize themselves for higher dimensional planar systems.

In the nervous system, noise is present almost everywhere and has great impact for brain functioning. It is essential to consider neuronal noise as an additional source of information [5]. Mandell et al. [6] explored slow periodic fast colour noise driving the neuronal dynamics in the neocortex. The dynamical phenomenon which they studied is known as "*Neocortical Resonance*" . The spiking bursting behavior of neural systems is a multi-time-scale activity which is caused by the slow process i.e., bursting modulating the fast firing activity i.e., spiking. Certain types of neurons in different regions of brain such as neurons in the thalamus region during drowsiness and sleep produce bursting [7]. It can be observed from electroencephalography recordings that the oscillatory patterns during sleeping of humans are considered as the synchronized firing of spiking bursting activity of neurons. Some neurological disorders, such as Parkinson's disease, epilepsy and schizophrenia have been found to be originated from some variances in the activity of some specified neurons in certain brain areas [8,9]. The effects of different stimulation paradigms on the firing rates of Parkinsonism were examined using the conductance based dynamical models [10].

The synchronization analysis of spiking and bursting from biophysical neural systems may help us to understand the activity for signal processing in neurons and even the origins of some neurological diseases. The synchronized activity of coupled neural systems may involve synchronization of individual spikes or bursting or both. In coupled systems, the oscillatory behavior become less incoherent and finally nearly or completely coherent with the increase of coupling strengths. The multiple spikes for bursting or

* Corresponding author.
E-mail addresses: ranjit_ism@yahoo.com, ranjit.chaos@gmail.com (R.K. Upadhyay), argham99@gmail.com (A. Mondal).

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individual spikes for spiking are synchronized in phase and then synchronized in amplitude appeared.

Izhikevich [2] has considered a single neuron model with slow variations in the applied d. c. current. The paper explores dynamical consequences of synaptic and electrical coupling in noisy environment in the same model. Synchronized neuronal activity presents neural signal processing, coding and such a behavior for neural network. The presence of synchronous behavior has been often found in excitable neural cells [8,9,11,12], pancreatic β-cells [13] and in many non excitable neurons. Bursting [2,14] presents two states activities between repetitive spiking and quiescence. It is responsible for two important aspects: one for information processing between neurons and signal transmission, other for neuronal coupling and synchronization [2,15]. Bursting can occur in single cell and also in network of cells. It is a multi-scale-type phenomenon where spiking, bursting or both can exist.

The M-L model exhibits neuro-computational features of real neurons and have significant importance in information transmission and temporal coding. It can exhibit different types of spiking and bursting using external injected current at various parameter sets. The above neural model presents tonic spiking and bursting. It is both class I and class II excitable [16,17]. It has already been observed from bifurcation analysis that some bursters are integrators and other resonators. Integrator may show burst synchronization and resonators may exhibit both types of spike and burst synchronization.

Generally, synaptic noise generates from spiking or bursting in the molecular events followed by the components of synaptic terminal. Chemical synapses release the neurotransmitters probabilistically. It often releases the molecular components at a very low rate in the absence of spikes. The release rate depends on the firing activity of both connected neurons. The fluctuation in the synaptic strength and connection may noisy injection of stimulus current in the cell. The spikes coming to the synapses release a small random amount of charges in the neuron due to the synaptic release noise. The post synaptic neuron has some random stimulus fluctuations due to the effect of pre-synaptic firing activity of spikes. The irregular influences of the synaptic release noise ranges from random activity to periodic or oscillatory inputs. It strongly acts to the in vivo experimental studies. It increases the mean conductances values of the cell. The influences of synaptic noise on the firing activity governs the firing times and strong noise intensity effects on the dynamics of the firings [5,14,18].

In this paper, we have studied synchronization and its stability by introducing a computationally efficient coupled M-L systems. We consider chemical and electrical coupling in noisy environment to explore synchronization by varying coupling strengths in a specified range and show that appropriate choice of noise strengths can effect the phase and amplitude of the coupled neural systems. Bursting controls the transmission failure through synapse and it is more important than single spiking for the generation of synchronization. These types of bursting are largely used in the simulation of the cortical neural networks [2,19]. The two parameter sets are presented for point-cycle type bursting pattern. With the increase of coupling strengths, it increases the locking between neurons (anti-phase for inhibitory case and in phase for excitatory case). The strong noise amplitudes destroy the original bursting pattern and synchronous activity of coupled neurons.

2. Spiking bursting single neural models

Two-variable neural model [20] does not support bursting. A third variable must be added. The applied current can be assumed to be slow varying so that it is the slowest among the three system variables. The system represents biological characteristics of neurons which are in interaction with a given environment represented by injected current to the system externally. The following set of coupled ordinary differential equations (ODEs) describes the bursting with variables (x, y) defined as the fast subsystem and variable z the slower one [2].

$$\frac{dx}{dt} = z - g_{Ca} m_\infty(x)(x - V_{Ca}) - g_K y (x - V_K) - g_L (x - V_L), \tag{1a}$$

$$\frac{dy}{dt} = (w_\infty(x) - y)/\tau_w(x), \tag{1b}$$

$$\frac{dz}{dt} = -\mu(V_0 + x), \tag{1c}$$

where $m_\infty(x) = 0.5(1 + \tanh((x - V_1)/V_2))$, $w_\infty(x) = 0.5(1 + \tanh((x - V_3)/V_4))$ and $\tau_w(x) = [\phi \cosh((x - V_3)/(2V_4))]^{-1}$. The aforementioned model consists of the voltage-gated Ca^{+2} current, voltage-gated delayed rectifier K^+ current and the leakage current. The variable x represents the membrane potential of the neuronal cell, y is the activation variable for the K^+ ion channels and z is the external input current. The parameters g_{Ca}, g_K and g_L represent the maximum conductances corresponding to Ca^{+2}, K^+ and leak currents respectively. V_K and V_L represent the reversal potential corresponding to the above ionic currents. The parameter ϕ represents the temparature scaling factor for the K^+ channel opening. The parameters V_1, V_2, V_3 and V_4 are appropriately considered for the hyperbolic functions so that they can attain their equilibrium points instantaneously. The value of μ is less than 1 i.e., $0 < \mu < 1$ [2,21].

2.1. The fast – subsystem

In original M-L system, the applied d. c. current to the neuronal cell is kept constant. Later it was explored the dynamics of this system when the external current is varied at a very slow rate and thus, considered as a state variable in the model [2,14,19]. The interaction of the fast M-L system and the slow subsystem yields a dynamics rich in several types of bursting which originates from bifurcations of the type fold/subHopf hysterisis loop of point-point and subHopf/fold cycle hysterisis loop [2].

2.2. The slow – subsystem

The applied current z is not constant, instead decays exponentially with a rate which is decided by both the current value of the membrane potential as well as factors external to the neural cell under study. The external factor may be regarded as representation of the fact that the cell always interacts with its environment. In single cell experiments, neuronal dynamics has been studied in isolation whereas in reality it is always a part of a subnetwork of larger network.

First, we present the phase plane analysis of the fast-slow system (1a)–(1c) at various set of parameter values namely as bursting neuron 1 and 2 (see Fig. 1). It has been observed that the geometry of the attractors as well as the dynamics of the system changes with changing the control parameters. Then, we study the synchronous behavior of the two coupled identical as well as non-identical bursting M-L neurons in Section 3.

3. Dynamics of coupled M-L systems

In this section, we first describe the two types of synaptic coupling scheme in presence of noise. Next, the synchronization analysis of coupled identical as well as non-identical M-L neurons are studied at two parameter sets (Case I and II respectively). The stability of synchronization is measured using a dynamical analysis known as Similarity function. We derived the necessary condition