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Evaluation of the impact of higher-order energy enhancement characteristics of solitons in strongly dispersion-managed optical fibers

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ABSTRACT

We study the propagation properties of nonlinear pulses with periodic evolution in a dispersion-managed transmission link by means of a variational approach. We fit the energy enhancement required for stable propagation of a single soliton in a prototypical commercial link to a polynomial approximation that describes the dependence of the energy on the map strength of the normalized unit cell. We present an improvement of a relatively old and essential result, namely, the dependence of the energy-enhancement factor of dispersion-management solitons with the square of the map strength of the fiber link. We find that adding additional corrections to the conventional quadratic formula up to the fourth order results in an improvement in the accuracy of the description of the numerical results obtained with the variational approximation. Even a small error in the energy is found to introduce large deviations in the pulse parameters during its evolution. The error in the evaluation of the interaction distance between two adjacent time division multiplexed pulses propagating in the same channel in a prototypical submarine link is of the same order as the error in the energy.

1. Introduction

In recent years, solitons have been an active research area in physics and mathematics and have played an important role in condensed matter physics and nonlinear optics [1–3]. In nonlinear optics many local nonlinear media structures have been extensively studied, such as spatial and spatiotemporal solitons [4], vortex solitons, light bullets and breathers [5], with applications in dense wavelength division multiplexing, soliton supercontinuum generation, new soliton lasers design, etc.

It is well known that soliton pulses have excellent properties as information carriers in high bit-rates and long distance optical transmission links both for time-division and wavelength-division multiplexing systems (TDM and WDM, respectively) [6]. Optical soliton communication relies on the exact balance between the group velocity dispersion (GVD) and the self-phase modulation (SPM) of the pulse along the transmission link. When combined with the Dispersion Management (DM) technique, consisting on a periodic alternation of fiber segments with normal and anomalous dispersion, the transmission distance of these kind of optical transmission systems can reach several thousand kilometers and their bit rates tenths of Gb/s [7]. This advantage comes from the fact that there is an energy enhancement compared with that of the conventional soliton [8–11] that raises the signal-to-noise ratio (SNR) and reduces the noise induced timing jitter and other beneficial features of DM, such as the reduction of the impact of four wave mixing (FWM) effects. Unlike constant dispersion networks, where there is no change in the pulse shape, in DM networks the pulse width oscillates periodically and the shape of the pulse ranges from hyperbolic secant to Gaussian, depending on the strength of the dispersion management, defined as the difference between normal and anomalous values of the local dispersion ΔD [7]. In general, optical fiber links with strong dispersion management ($10 < Z_0 \Delta D < 45$), where the pulse is well defined by means of a Gaussian ansatz, combined with fiber segments with positive GVD-sign, are one of the most promising candidates for next generation telecommunication networks. However, there are several system penalties associated with DM propagation even in a single frequency channel. One of the most important arises from a large pulse stretching associated with the increase in the pulse energy and breathing within one dispersion period accompanied with an overlap of neighboring pulses that causes a reduction in the interaction distance [12]. And as we are moving towards narrower pulse widths, up to picosecond and even femtosecond time slots, higher-order dispersion effects, such as third-order dispersion (TOD) and nonlinearity, must be considered. Both of them determine a significant enhancement of the Gordon–Haus effect and the timing jitter and also induce crosstalk [13,14]. Due to all these restrictions,

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a precise knowledge of the initial pulse energy is critical to determine stable pulse transmission in the design of a DM optical link.

In this work, we consider polynomial higher-order approximations for the improved accuracy description of the power enhancement factor of single optical soliton pulses as a function of the physical characteristics of the optical fiber link. Previous well known works have used a quadratic formula for the power enhancement based on the second order path averaged Group Velocity Dispersion (GVD) and the initial pulse energy of a soliton of equal full width half-maximum (FWHM) in a uniform fiber with the same path-average dispersion and nonlinear coefficient [9]. This formula has been used extensively to examine the asymptotic limit where the dispersion-map period is shorter than the fiber's nonlinear length scale [15], to obtain the degree to which the pulse energy must be enhanced to maintain any given initial pulse width [16], to model the interactions in optical fibers [17] and for different amplifier locations inside the dispersion map in lossy systems [18,19]. Now, we analyze the approximation error in this type of polynomial expression as new higher-order terms are introduced in the Generalized Nonlinear Schrödinger Equation (GNLS) for a specific and prototypical commercial submarine network. The results shows that at fourth order a good compromise between accuracy and complexity can be obtained with an impact on the estimation of the interaction distance between two solitons of the same order of magnitude. The impact on the pulse inverse width, on the other hand, can be very large.

The paper is arranged as follows. In Section 2, we study the mathematical model that describes the propagation of soliton pulses through the well-known Generalized Nonlinear Schrödinger Equation (GNLS) and we obtain the reduced ordinary differential equations (ODE) model by means of a variational method [20] which takes into account third order dispersion [21]. In Section 3, we compare the DM single soliton energy required to obtain stable pulse propagation obtained through the variational approximation with that obtained when taking into account higher order scaling corrections in the classical quadratic formula that describes the dependence of the energy enhancement factor with the map strength. In Section 4, using the ODEs obtained through the variational approximation, we extend the analysis and compare the interaction distances of two simultaneously propagating pulses with initial energy obtained in the ODE model, with those distances obtained when the values of energy obtained with higher-order correction terms in the analytical equation are considered. Finally, in Section 5, we present the conclusions of our work.

2. Ordinary differential equations model

We analyze a DM optical fiber made up of alternating segments of equal length with normal and anomalous dispersion. In our analysis, we consider a lossless time-division multiplexed system and study a couple of adjacent pulses propagating in the same channel. We employ a well-known variational method [20,22] which permits to reduce the full complexity of the GNLS to that of a system of ODEs which capture the most relevant features of the evolving solutions in an approximate manner. We assume, as it happens in the strong management regime, that the interacting pulses are well approximated by a Gaussian shape [23,24] and use the ansatz

$$u_l(Z,T) = \sqrt{\frac{E_l}{\sqrt{\pi}}}\sqrt{p_l(Z)}\exp\left[\frac{-p_l(Z)^2}{2}(1-jC_l(Z))(T-T_l(Z))^2 - j\omega_l(Z)(T-T_l(Z)) + j\theta_l(Z)\right]. \tag{1}$$

where E_l, $p_l(Z)$, $C_l(Z)$, $\omega_l(Z)$, $T_l(Z)$, $\theta_l(Z)$ are the energy, inverse pulse width, linear chirp, center frequency, center position and phase of the pulse, respectively. In order to address the interaction properties of intrachannel DM solitons in TDM transmission systems, we consider a solution consisting of 2 interacting pulses $u(Z, T) = u_1(Z, T) + u_2(Z, T)$, resulting the system of equations for the evolution of the pulses

$$j\frac{\partial u_l}{\partial Z} + \frac{1}{2}D(Z)\frac{\partial^2 u_l}{\partial T^2} + S(Z)|u_l|^2 u_l + 2S(Z)\sum_{m\neq l}|u_m|^2 u_l - j\delta\frac{\partial^3 u_l}{\partial T^3} = 0,$$
$$l = 1, 2. \tag{2}$$

$D(Z)$ defines the dispersion map and it is a periodic function of the propagation distance with alternating values D_+ and D_- in sections of fiber with lengths Z_+ and Z_-. $\Delta D = D_+ - |D_-|$is the dispersion difference and $Z_0 = Z_+ + Z_-$ is the map period.

In our mathematical model, we neglect the phase-dependent terms. That is, we are considering an incoherent model where the interaction effects are dictated by cross-phase modulation (XPM) and self-phase modulation terms (SPM) in what is called the strong management regime ($Z_0\Delta D > 10$, $Z_0\Delta D < 45$). As transmission bit rates move to higher standards of the synchronous optical network (SONET) or the synchronous digital hierarchy (SDH), the impact of higher-order effects becomes increasingly important. For such cases, third-order dispersion (TOD) must be taken into account. In our results we assume a constant value for the TOD parameter $\delta = sign\,(d^3\beta/d\omega^3)/6$ that models the effect of TOD stemming from the term $d^3\beta/d\omega^3$, where $\beta(\omega)$ is the mode propagation constant at frequency ω. TOD induces three main effects in DM soliton propagation: a displacement of the pulse position, an asymmetric distortion in the shape of the pulse and energy radiation. We have neglected the two latter effects in the variational approximation. This restricts its validity to small values of δ. It may seem a limitation of the proposed variational model but, on the other hand, this condition defines a regime of practical interest for a transmission system where TOD-induced radiation loss has to be kept to a very small value. It has been previously reported [25] that these effects induced by TOD are negligible for typical system parameters.

Substituting the ansatz (1) in the Lagrangian density from which (2) is obtained and integrating in the transverse coordinate T permits to obtain the Lagrangian for the reduced dynamical system. Finally, taking the variation with respect to each of the pulse parameters p_l, C_l, ω_l, T_l, we obtain the equations of motion

$$\frac{dp_l}{dZ} = -C_l {p_l}^3(D - 6\delta\omega_l) \tag{3}$$

$$\frac{dC_l}{dZ} = (1 + {C_l}^2){p_l}^2(D - 6\delta\omega_l) - \frac{p_l S(Z)}{\sqrt{2\pi}}\left(E_l + \frac{2}{{p_l}^3}\sum_{m\neq l}E_m P_{l,m}^3(1 - {\tau_{l,m}}^2)\exp\left(\frac{-{\tau_{l,m}}^2}{2}\right)\right). \tag{4}$$

$$\frac{dT_l}{dZ} = -D\omega_l + 3\delta{\omega_l}^2 + \frac{3}{2}\delta(1 + {C_l}^2){p_l}^2. \tag{5}$$

$$\frac{d\omega_l}{dZ} = \frac{2S(Z)}{\sqrt{2\pi}}\sum_{m\neq l}E_m\tau_{l,m}{P_{l,m}}^2\exp\left(\frac{-{\tau_{l,m}}^2}{2}\right) \tag{6}$$

Where

$$\tau_{l,m} = P_{l,m}\Delta T_{l,m} \tag{7}$$

$$\Delta T_{l,m} = T_l - T_m \tag{8}$$

$$P_{l,m} = \frac{\sqrt{2}p_l p_m}{\sqrt{{p_l}^2 + {p_m}^2}} \tag{9}$$