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New aspects of the adaptive synchronization and hyperchaos suppression of a financial model

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ABSTRACT

This paper mainly focuses on the analysis of a hyperchaotic financial system as well as its chaos control and synchronization. The phase diagrams of the above system are plotted and its dynamical behaviours like equilibrium points, stability, hyperchaotic attractors and Lyapunov exponents are investigated. In order to control the hyperchaos, an efficient optimal controller based on the Pontryagin's maximum principle is designed and an adaptive controller established by the Lyapunov stability theory is also implemented. Furthermore, two identical financial models are globally synchronized by using an interesting adaptive control scheme. Finally, a fractional economic model is introduced which can also generate hyperchaotic attractors. In this case, a linear state feedback controller together with an active control technique are used in order to control the hyperchaos and realize the synchronization, respectively. Numerical simulations verifying the theoretical analysis are included.

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1. Introduction

Chaos is one of the most interesting phenomena which has received a growing attention owing to its potential applications in various fields including finance [1] and economics [2]. However, the chaotic behaviour in the financial systems should be diminished in order to improve the economic performance. Motivated by this background, modeling, control and synchronization of chaotic/hyperchaotic financial systems have been active topics of study to be considered by many researchers. In [3], the chaotic behaviour of an economical model was controlled by using a time-delayed feedback control scheme. In [4], the chaos phenomenon in a Cournot duopoly model was controlled by employing an adaptive parameter-tuning strategy. In a Cournot duopoly model [5], the chaos was reduced by using a minimum entropy algorithm. In a Behrens-Feichtinger model [6], a minimum entropy strategy was employed to diminish the chaos through a delayed feedback. In [7], chaos control in economic models was achieved by applying a straight-line stabilization technique. A chaotic system describing the FDI in China was controlled by using a time-delayed feedback method [8]. In [9], some effective feedback controllers have been designed for stabilizing hyperchaos to unstable equilibrium points. An efficient adaptive control scheme for economic models has been investigated in [10]. Based on the Lyapunov stability theory, an adaptive algorithm was employed in [11] in order to synchronize the chaotic financial systems. In [12], an adaptive control technique was presented for a hyperchaotic financial system which guarantees the asymptotic stability of the synchronization error.

Fractional calculus is developing fast and its various applications are extensively used in many fields of science and engineering [13–16]. Many scientists have demonstrated that the fractional order representation provides more realistic behaviours of many phenomena in various fields [17,18]. However, the fractional-order models can exhibit chaotic/hyperchaotic behaviours which should be controlled [19–22]. Recently, the fractional modeling in the life science and economy has gained much attention [23–25]. Recent papers studying the chaos control and synchronization of fractional economic systems applied various control methodologies such as active control [26], sliding mode control [27], etc. The main problem in applying such schemes is often due to their complexity in implementation.

The properties of the chaotic/hyperchaotic financial systems should be deeper investigated and efficient methods should be continuously extended to control the chaotic/hyperchaotic behaviours of the economic models. Inspired by the above discussions, this paper investigates the dynamical behaviours of a hyperchaotic financial system such as its equilibrium points, stability,

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hyperchaotic attractors and Lyapunov exponents. Moreover, an optimal controller is designed according to the Pontryagin's maximum principle in order to stabilize the hyperchaos to unstable equilibrium points. Furthermore, the adaptive controllers are derived for the stabilization and synchronization of the given economic model with unknown parameters. The stability condition is obtained in both of theoretical analysis and simulation manner. Finally, it is shown that the financial system in fractional sense can also generate hyperchaotic attractors. In this case, a linear state feedback control is derived for stabilizing the hyperchaos to unstable equilibrium points and an active controller is used to achieve synchronization between two identical fractional-order hyperchaotic systems. Numerical simulations verifying the theoretical results are given.

The outline of this paper follows here. In Section 2, a hyperchaotic financial system is considered and its fundamental properties are studied. This section also includes the hyperchaos control and synchronization results for the financial model under consideration. In Section 3, the fractional-order version of this model is presented. In this section, a linear state feedback controller and an active control technique are employed to stabilize hyperchaos and realize synchronization for the fractional economic model, respectively. Finally, we finish the manuscript by a conclusion part.

2. A hyperchaotic financial system

In 1993, Huang and Li [28] established a 3D chaotic economic model which is composed of four sub-blocks: labor force, stock, money and production. This chaotic financial system is expressed by

$$\begin{cases} \dot{f}_1 = f_3 + (f_2 - \alpha) f_1, \\ \dot{f}_2 = 1 - \beta f_2 - f_1^2, \\ \dot{f}_3 = -f_1 - \gamma f_3, \end{cases} \tag{1}$$

where f_1, f_2 and f_3 are the interest rate, investment demand and price index, respectively. Moreover, the parameters $\alpha \geq 0$, $\beta \geq 0$ and $\gamma \geq 0$ denote the amount of savings, investment cost and demand elasticity of commodity, respectively. The changes in f_1 are influenced by the structural adjustment of the prices and the contradictions of the investment market. The changing rate of f_2 is proportional to an inversion with the cost of investment and interest rates. It is also proportional to the rate of investment. The contradiction between supply and demand in commercial markets controls the changes in f_3. These changes are also affected by the inflation rates. The dynamical behaviours of the financial system (1) have been investigated by the researchers [29]. The global complicated character of this model as well as its bifurcation topological structure have been studied by the authors in [30,31]. The system (1) in fractional sense has been analyzed in [23]. A discrete form of the financial model (1) and its Neimark-Sacker bifurcation have been studied in [32]. The uncertain fractional and stochastic forms of system (1) were also presented in [33] and [34], respectively. In order to realize the chaos encryption in higher dimension, the system (1) was modified in [35] to construct a new 4D hyperchaotic system. Following the same idea as in [35], here we add a new state variable f_4 called the average profit margin to the economic model (1). Thus, the 4D financial system is constructed as

$$\begin{cases} \dot{f}_1 = f_3 + (f_2 - \alpha) f_1 + f_4, \\ \dot{f}_2 = 1 - \beta f_2 - f_1^2, \\ \dot{f}_3 = -f_1 - \gamma f_3, \\ \dot{f}_4 = -0.05 f_1 f_3 + \delta f_4, \end{cases} \tag{2}$$

where δ is a constant parameter.

The chaos in economics expresses the inherent instability in macroeconomics. The chaotic behaviour can potentially explain fluctuations in financial markets and economy which appear to be random. Thus, we will consider the system parameters in Eq. (2) in such a way that the economic model (2) exhibits hyperchaotic behaviour. In the following, we will show that the given system by Eq. (2) behaves in a hyperchaotic manner when $(\alpha, \beta, \gamma, \delta) = (0.9, 0.1, 1, -0.6)$. To this end, we use the Wolf algorithm [36] in order to compute the Lyapunov exponents of the proposed system

$$\begin{aligned} L_1 &= 0.100039, \\ L_2 &= 0.003239, \\ L_3 &= 0.488120, \\ L_4 &= -0.621555. \end{aligned} \tag{3}$$

From Eq. (3) it is observed that $L_1, L_2, L_3 > 0$ and $\sum_{i=1}^{4} L_i < 0$. Furthermore, the Lyapunov dimension of system (2) is calculated by the Kaplan-Yorke conjecture

$$\begin{aligned} D_{KY} &= 2 + \frac{L_1 + L_2}{|L_3|} \\ &= 2 + \frac{0.100039 + 0.003239}{|-0.488120|} = 2.211583, \end{aligned} \tag{4}$$

which is fractional. Consequently, the system given by Eq. (2) exhibits hyperchaotic behaviour. For the system parameters $(\alpha, \beta, \gamma, \delta) = (0.9, 0.1, 1, -0.6)$ and the initial states $(f_1(0), f_2(0), f_3(0), f_4(0)) = (-0.1, 2.5, -4, 5)$, the corresponding hyperchaotic attractors are depicted in Fig. 1. More basic properties of this system will be given in the next section.

2.1. Qualitative analysis and dynamical behaviours

This section discusses the dynamical behaviours of the 4D hyperchaotic economic model (2).

2.1.1. Dissipation and hyperchaotic attractor existence

In accordance with Eq. (2), if we consider the vector field L as

$$L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} = \begin{bmatrix} f_3 + (f_2 - \alpha) f_1 + f_4 \\ 1 - \beta f_2 - f_1^2 \\ -f_1 - \gamma f_3 \\ -0.05 f_1 f_3 + \delta f_4 \end{bmatrix}, \tag{5}$$

then the divergence of L is easily calculated from

$$\nabla . L = \frac{\partial l_1}{\partial f_1} + \frac{\partial l_2}{\partial f_2} + \frac{\partial l_3}{\partial f_3} + \frac{\partial l_4}{\partial f_4} = -(\alpha + \beta + \gamma - \delta). \tag{6}$$

The system (2) is dissipative if and only if the divergence of L is negative. Thus, in accordance with Eq. (6), the economic model (2) is a dissipative system if and only if the condition $\alpha + \beta + \gamma - \delta > 0$ is satisfied. Under this condition, each volume containing the trajectories of the dynamical system (2) shrinks to zero at an exponential rate $\alpha + \beta + \gamma - \delta$. Thus, all the orbits are ultimately limited to a special subset of zero volume, and all trajectories of the considered system evolve to an attractor set as $t \to \infty$.

Remark 2.1. In [37], Caputo has shown that the memory effects of the real processes also lead to dissipation. As the financial systems have memory effects (the past economic behaviour may affect the present and future ones), the dissipative property of the economic processes can also be connected with their memory.

2.1.2. Equilibrium points and stability

By a simple calculation, it can be shown that when $(\alpha, \beta, \gamma, \delta) = (0.9, 0.1, 1, -0.6)$, the financial system (2) has three equilibrium points

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