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Impulsive fractional differential equations with Riemann–Liouville derivative and iterative learning control☆

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ABSTRACT

We try to seek a representation of solution to an initial value problem for impulsive fractional differential equations (IFDEs for short) involving Riemann–Liouvill (RL for short) fractional derivative, then prove an interesting existence result, and introduce Ulam type stability concepts of solution for this kind of equations by introducing some differential inequalities. In addition, we study iterative learning control (ILC for short) problem for system governed by IFDEs via a varying iterative state that does not coincide with a given initial state and apply proportional type learning principle involving the original learning condition to generate each output to following the final path in a finite time interval, then give a convergence result. Numerical examples are reported to check existence and stability of solutions and display the error for different iterative times.

1. Introduction

Fractional order calculus is a generalization of the integer order calculus, which goes back to the 17th-century mathematicians' era Leibniz and L'Hôpital. For the last several years, there are a lot of monographs and papers on the background and application of fractional calculus [1–6] and fundamental theory (existence, stability, control theory) for all kinds of fractional differential equations and inclusions [7–19], which are originated from the issues in physics, control engineering and so on.

The classical impulsive differential equations have been used to formulated the mathematics modelling for the real problems raised from biology and economic. Experiences in physics show that whether Hooke's law or Newton's law does not to describe the dynamic behavior well in viscous liquids. Note that a mass-spring-damper system with impulsive effects is established to deal with the behavior well appearing in viscous liquids subjected to earthquake at some certain time. Existence theory of solutions via control theory for IFDEs [20–26] have been attracted more and more mathematicians as well as physicists.

Stability is always an interesting topic for IFDEs, a small number of results [27–30] since the standard Lyapunov method is facing more challenge due to the nonlocal property of fractional derivative. A special stability problem about Cauchy equation associated with concept on group homomorphisms was put forward for the first time in 1940 by Ulam. Thereafter, Hyers solves this problem partly [31]. In 1978, Rassias [32] obtained a result for the above problem. In addition, Rus [33] presented various types of Ulam-type stability for ODEs. Note that an IFDE can be turned to an equation with integral in some sense. Thus, motivated by Rus [33], Wang et al. [34,35] introduced Ulam's stability concepts for impulsive ODEs, which provide an alternative way to study stability of impulsive dynamic systems from different points of view.

ILC is widely used in industry, such as robot system, industrial control process, inverted pendulum control and so on. Recently, there are some interesting results for designing ILC update laws for various of differential systems not limited in the classical integer order differential systems. For example, ILC for fractional dif-

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ferential equations [36–40], ILC for impulsive differential systems [41–44], and so on.

Now we study

$$\begin{cases} (\mathscr{D}^{\alpha}_{0^+}X)(t) = F(t, X(t), U(t)), \ t \in V' := V \setminus \{\sigma_i\}_{i=0}^{m}, \\ \quad V = (0, T], \ \alpha \in (0, 1), \\ (\mathscr{I}^{1-\alpha}_{0^+}X)(0^+) = X(0), \\ (\mathscr{I}^{1-\alpha}_{0^+}X)(\sigma_j^+) = (\mathscr{I}^{1-\alpha}_{0^+}X)(\sigma_j^-) + W_j, \ \sigma_j \in \Lambda := \{\sigma_i\}_{i=0}^{m}, \end{cases} \tag{1}$$

and

$$\begin{cases} (\mathscr{D}^{\alpha}_{0^+}X_k)(t) = F(t, X_k(t), U_k(t)), \ t \in V', \\ (\mathscr{I}^{1-\alpha}_{0^+}X_k)(0^+) = X_k(0), \\ (\mathscr{I}^{1-\alpha}_{0^+}X_k)(\sigma_j^+) = (\mathscr{I}^{1-\alpha}_{0^+}X_k)(\sigma_j^-) + W_j, \ \sigma_j \in \Lambda, \\ \mathscr{Y}_k(t) = \mathscr{G}(t, X_k(t)) + \mathscr{B}U_k(t), \end{cases} \tag{2}$$

where $\mathscr{D}^{\alpha}_{0^+}$ denotes RL-fractional derivative and $\mathscr{I}^{1-\alpha}_{0^+}$ denotes RL-fractional integral (see Definition 2.1), k denotes the kth learning iteration, $T > 0$ is given iteration domain length. $\mathscr{I}^{1-\alpha}_{0^+}X(\sigma_j^{\pm}) = \lim_{\varepsilon\to 0^{\pm}} \mathscr{I}^{1-\alpha}_{0^+}X(\sigma_j \pm \varepsilon)$ and $\mathscr{I}^{1-\alpha}_{0^+}X(\sigma_j^-) = \mathscr{I}^{1-\alpha}_{0^+}X(\sigma_j)$, see [8, Lemma 3.2, Chapter 3]. And σ_j, $j = 1, 2\cdots m$ denotes the jth impulsive points satisfying $\sigma_j < \sigma_{j+1}$, $j = 0, 1, \ldots, m$ with $0 = \sigma_0$ and $\sigma_{m+1} = T$. The nonlinear terms $F(\cdot,\cdot,\cdot) \in C([0, T] \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)$ such that $F(\cdot, X, U) \in C_\gamma([0, T], \mathbb{R}^n)$ for any $X, U \in \mathbb{R}^n$ and $\mathscr{G} \in C(V \times \mathbb{R}^n, \mathbb{R}^n)$ are given functions. The variables X_k, $\mathscr{Y}_k$, $W_i \in \mathbb{R}^n$ are denoted by state, output and impulse term respectively. The control function $U_k \in L^q([0, T], \mathbb{R}^n)(q > \frac{1}{\alpha})$. Moreover, $\mathscr{B}$ is $n \times n$ real constant matrix.

Following [21,22,26], we seek a possible representation of solutions of Cauchy problems for (1) in Section 2 and give existence, uniqueness result in Section 3 and introduce the associated Ulam's stability concepts for (1) by virtue of some desired impulsive differential inequalities in Section 4. Next, ILC problem for (2) is considered in Section 5. We generate a sequence of controls $U_k(t)$ enables $\mathscr{Y}_k(t)$ to track the given reference trajectory $\mathscr{Y}_d(t)$ (maybe continuous or discontinuous) precisely enough when k is sufficiently large on finite time interval [0, T] measured by (PC_γ, λ)-norm via designing proportional-type learning law with a deviation of the initial state. Examples are reported in final section.

About the novelty of this paper, we emphasize that we initially give a new concept of piecewise continuous solutions of (1), introduce Ulam type stability concepts and analyze existence and stability of solutions, which fits into recently developing theory of IFDEs with Caputo and Hadamard derivatives [21,22,26]. The new obtained concept of piecewise continuous solutions of (1) can display that fractional order derivative has global property and focus on the memory accumulated by the long-term effects in the whole evolution process including some fixed impulsive moments, which is much different from the previous problem arises from the classical impulsive ODEs. It is well known that impulsive controlled systems can generate a sequences of local continuous state trajectories that will be used to seek the desired local continuous trajectories in the experience of robotic fish with additional jet enginery at some fixed time. Following this idea we extend the results in Liu et al. [42], Yu et al. [43] to RL fractional derivative case based on the previous qualitative theory analysis basis.

2. Preliminaries

Set $\overline{V} = [0, T]$ and $N_m := \{1, 2, \ldots, m\}$. Let $C(\overline{V}, \mathbb{R}^n)$ be the space of vector-value continuous functions from $\overline{V} \to \mathbb{R}^n$ endowed with $\|X\|_C = \sup_{t\in\overline{V}} \|X(t)\|$ where $\|\cdot\|$ denotes a standard vector norm. For $\gamma \geq 1 - \alpha$, let $C_\gamma(\overline{V}, \mathbb{R}^n) = \{X(\cdot) : \cdot^{\gamma}X(\cdot) \in C(\overline{V}, \mathbb{R}^n)\}$ with $\|X\|_{C_\gamma} = \sup_{t\in\overline{V}} t^{\gamma}\|X(t)\|$. Obviously, $C_\gamma(\overline{V}, \mathbb{R}^n)$ is a Banach space.

Denote $PC_\gamma(\overline{V}, \mathbb{R}^n) = \{X(\cdot) : \cdot^{\gamma}X(\cdot) \in C((\sigma_j, \sigma_{j+1}], \mathbb{R}^n)$, and $\lim_{t\to\sigma_j^+} t^{\gamma}X(t)$ exists, $j \in N_m\}$ endowed with PC_γ-norm $\|X\|_{PC_\gamma} = \max_{j\in\{0,1,\ldots,m\}} \sup_{t\in(\sigma_j,\sigma_{j+1}]} t^{\gamma}\|X(t)\|$. Further, we introduce (PC_γ, λ)-norm which will be used in next sections $\|X\|_{PC_{\gamma,\lambda}} = \max_{j\in\{0,1\ldots,m\}} \sup_{t\in(\sigma_j,\sigma_{j+1}]} e^{-\lambda t}t^{\gamma}\|X(t)\|$. Specially, $\lambda = 0$, (PC_γ, λ)-norm is PC_γ-norm.

Definition 2.1 (see [8, Formulas (2.1.1), (2.1.5)]). For an integrable function g, the RL fractional integral $\mathscr{I}^{\alpha}_{c^+}g(1 > \alpha > 0)$ is $(\mathscr{I}^{\alpha}_{c^+}g)(z) := \int_c^z \frac{g(s)}{\Gamma(\alpha)(z-s)^{1-\alpha}}ds$, $c < z$, and the RL fractional derivative $\mathscr{D}^{\alpha}_{c^+}g$ is $(\mathscr{D}^{\alpha}_{c^+}g)(z) := \frac{d}{dz}(\mathscr{I}^{1-\alpha}_{c^+}g)(z) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dz}\int_c^z \frac{g(t)}{(z-s)^{\alpha}}ds$.

Definition 2.2 (see [9, (4.1.1)]). The function $E_c(\nu) := E_{c,1}(\nu) = \sum_{k=0}^{\infty} \frac{\nu^k}{\Gamma(ck+1)}$, $c > 0$, $\nu \in \mathbb{R}$, is called a function of Mittag–Leffler with one parameter.

Lemma 2.3 (see [8, Property 2.1]). *If $c \geq 0$, $d > 0$, then* $(\mathscr{I}^{c}_{r^+}(t-r)^{d-1})(x) = \frac{\Gamma(d)}{\Gamma(d+c)}(x-r)^{d+c-1}$, $(\mathscr{D}^{c}_{r^+}(t-r)^{d-1})(x) = \frac{\Gamma(d)}{\Gamma(d-c)}(x-r)^{d-c-1}$.

Lemma 2.4 (see [8, Lemma 2.3 and Lemma 2.4]). *If $c > 0$ and $d > 0$, then $(\mathscr{I}^{c}_{r^+}\mathscr{I}^{d}_{r^+}h)(x) = (\mathscr{I}^{c+d}_{r^+}h)(x)$, $(\mathscr{D}^{c}_{r^+}\mathscr{I}^{c}_{r^+}h)(x) = h(x)$ hold at almost every point $x \in \overline{V}$ for $h \in C_\gamma(\overline{V}, \mathbb{R}^n)$. If $c + d > 1$, then the first equality holds at any point of $\overline{V}$.*

Lemma 2.5 (see [35]). *Suppose $\vartheta \in PC(\overline{V}, \mathbb{R}^n)$ and*

$$\|\vartheta(\tau)\| \leq b_1(\tau) + b_2\int_0^{\tau} \frac{\|\vartheta(t)\|}{(\tau-t)^{1-d}}dt + \sum_{0<\sigma_k<\tau} \iota_k\|\vartheta(\sigma_k)\|, \ d \in (0, 1),$$

where $b_1(\cdot) \in C(\overline{V}, \mathbb{R}^+)$ is not decreasing on $\overline{V}$, b_2, $\iota_k > 0$ are constants. Then $\|\vartheta(\tau)\| \leq b_1(\tau)\left(1 + \iota E_d(b_2\Gamma(d)\tau^d)\right)^k E_d(b_2\Gamma(d)\tau^d)$, for $\tau \in (\sigma_k, \sigma_{k+1}]$, where $\iota = \max_{k\in N_m}\iota_k$.

3. Solutions of IFDEs

Concerning on

$$\begin{cases} (\mathscr{D}^{0.25}_{0^+}z)(t) = t, \ t \in (0, 1) \cup (1, 2], \ z(t) \in \mathbb{R}, \\ (\mathscr{I}^{0.75}_{0^+}z)(0^+) = 0, \\ (\mathscr{I}^{0.75}_{0^+}z)(1^+) = (\mathscr{I}^{0.75}_{0^+}z)(1^-) + 1. \end{cases} \tag{3}$$

According to [24, (2.5)], the solution of (3) is

$$z(t) = \begin{cases} \frac{t^{1.25}}{\Gamma(2.25)}, \ t \in (0, 1], \\ \frac{t^{1.25}}{\Gamma(2.25)} + \frac{(t-1)^{-3/4}}{\Gamma(1/4)}, \ 1 < t \leq 2. \end{cases} \tag{4}$$

However, by Lemma 2.3 and Definition 2.1, one derives

$$z(t) = \begin{cases} \frac{t^{1.25}}{\Gamma(2.25)}, \ t \in (0, 1], \\ \frac{t^{1.25}}{\Gamma(2.25)} + \frac{t^{-3/4}}{\Gamma(1/4)}, \ 1 < t \leq 2. \end{cases} \tag{5}$$

It is interesting for (3), (5) not coincides with (4) on $1 < t \leq 2$.

Now, we adopt the ideas in [21,26] to introduce another possible concept of solutions of (1).

Lemma 3.1. *Let $\varpi \in C(\overline{V}, \mathbb{R}^n)$. We say $Z \in C_\gamma(\overline{V}, \mathbb{R}^n)$ is a solution of*

$$\begin{cases} (\mathscr{D}^{\upsilon}_{0^+}Z)(t) = \varpi(t), \ t \in \overline{V}, \ 0 < \upsilon < 1 \\ (\mathscr{I}^{1-\upsilon}_{0^+}Z)(c) = Z_c, \ c > 0, \end{cases} \tag{6}$$