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Controllability of nonlinear stochastic fractional neutral systems with multiple time varying delays in control

R. Mabel Lizzy[a], Krishnan Balachandran[a], Juan J. Trujillo[b,*]

[a] Department of Mathematics, Bharathiar University, Coimbatore 641 046, India
[b] Departmento de Análisis Mathemático, Universidad de La Laquna, 38271 La Laguna, Tenerife, Spain

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ABSTRACT

Sufficient conditions for relative controllability of stochastic fractional neutral systems with bounded operator and multiple time varying delay in the control are obtained. The result is proved using an equivalent nonlinear integral equation to the system and Banach contraction principle. The controllability results are derived for systems with both Wiener and systems with Lévy noise.

1. Introduction

Controllability of nonlinear systems with time varying delays in input is essential to predict the true dynamics of the system. Most of the time presence of delays in the system is the main reason for deterioration of system performance and sometimes even instability [27]. The important relationships between stability and controllability of the systems [18] forms the motivation for the study of controllability property of systems with delay in the control variable. At the same time neutral differential equations are encountered in the description of various physical scenarios like the lossless transmission connection, stunted transmission connection [9], vibrating masses attached to an elastic bar [13] and collision problem in electrodynamics [12]. The controllability of neutral systems is studied by various authors, for instant [4,14]. At the same time incorporating fractional derivatives [15,17] and noises in mathematical models have proved to provide better understanding and approximations to real world processes. The controllability of deterministic systems and stochastic systems with time varying delay are dealt in [2] and [16] respectively. In this paper we study the controllability of stochastic fractional neutral systems with time varying multiple delay in infinite dimensional setting. The controllability concepts of deterministic and stochastic systems in infinite dimensions can be found in [3,6–8,10,24].The existing literature suggests two different ways of extending the controllability concepts to stochastic systems as follows

- The property of attaining all states in a suitable space of random variables, for example, the space of square integrable random variables.
- The property of attaining an arbitrarily small neighborhood of each point in the state space with probability arbitrarily near to one fortified with some uniformity

In the former approach which we adopt in this paper the state space consists of random variables whereas in the latter it consists of only nonrandom values see [21-23,25]. It is worth pointing out that most of the works on stochastic systems assume the noise process under discussion to be Wiener process. Unfortunately, the fluctuations in financial markets, sudden changes in the environment and many other real systems cannot be described by Brownian motion and this leads to the use of Lévy noise to model such discontinuous systems. This motivation makes the controllability problem of systems with other noise terms an interesting one. A detailed study of Lévy process in finite and infinite dimensions can

* Corresponding author.
E-mail addresses: mabel.math.bu@gmail.com (R.M. Lizzy), kb.maths.bu@gmail.com (K. Balachandran), jtrujill@ullmat.es (J.J. Trujillo).

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be found in [1,26] and the references therein. We intend to approach the controllability problem by providing an equivalent integral equation to the stochastic fractional neutral system with multiple time varying delay in control and use fixed point technique to arrive at the existence of control which steers the system to a desired random variable in a suitable state space. The controllability problem for stochastic fractional neutral system with multiple time-varying delay in control driven by both Wiener and Lévy noise are discussed here.

2. Preliminaries

Let X, U and K be separable Hilbert spaces and for convenience, we will use the same notation $\|\cdot\|$ to represent their norms. $\mathbb{L}(X, U)$ is the space of all bounded linear operators from X to U and J denotes the interval [0, T].

We assume that a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P})$ with the probability measure $\mathbf{P}$ on Ω, satisfying the "usual hypothesis":

(i) $\mathcal{F}_0$ contains all $A \in \mathcal{F}$ such that $\mathbf{P}$(A)=0,
(ii) $\mathcal{F}_t = \mathcal{F}_{t^+}$, $\forall t \in J$, where $\mathcal{F}_{t^+}$ is the intersection of all $\mathcal{F}_s$ where $s > t$, i.e., the filtration is right continuous.

Let us consider the following space settings. Denote,

- $Y := \mathbb{L}_2(\Omega, \mathcal{F}_T, X)$, which is the Hilbert space of all $\mathcal{F}_T$-measurable square integrable random variables with values in X.
- $\mathcal{H}_2$ is a closed subspace of $C(J, \mathbb{L}_2(\Omega, X))$ consisting of all $\mathcal{F}_t$-measurable processes with values in X, identifying processes which are modification of each other and endowed with the norm,

$$\|\phi\|^2_{\mathcal{H}_2} = \sup_{t\in J} \mathbf{E}\|\phi(t)\|^2,$$

where $\mathbf{E}$ denotes expectation with respect to $\mathbf{P}$.
- $U_{ad} := \mathbb{L}_2^{\mathcal{F}}(J, U)$, which is a Hilbert space of all square integrable and $\mathcal{F}_t$-measurable processes with values in U.
- $\mathcal{H}_2^0 := \mathbb{L}_2(\Omega, \mathcal{F}_0, X)$, which is the Hilbert space of all $\mathcal{F}_0$-measurable square integrable random variables with values in X.

Let us recall some basic definitions from fractional calculus. Let α, $\beta > 0$, with $n-1 < \alpha < n$, $n-1 < \beta < n$ and $n \in \mathbb{N}$. Suppose $f \in L_1(\mathbb{R}_+)$, $\mathbb{R}_+ = [0, \infty)$.

Definition 2.1 [17]**.** The Riemann Liouville fractional integral of a function f is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) \mathrm{d}s,$$

and the Caputo derivative of f is ${}^C D^\alpha f = I^{n-\alpha} D^n f$, that is,

$${}^C D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s) \mathrm{d}s,$$

where the function $f(t)$ has absolutely continuous derivative up to order $n-1$.

Definition 2.2 [17]**.** Let A be a bounded linear operator, the Mittag–Leffler operator function is given by,

$$E_{\alpha,\beta}(A) = \sum_{k=0}^{\infty} \frac{A^k}{\Gamma(k\alpha+\beta)}.$$

In particular, for $\beta = 1$,

$$E_{\alpha,1}(A) = E_\alpha(A) = \sum_{k=0}^{\infty} \frac{A^k}{\Gamma(k\alpha+1)}.$$

Consider the linear stochastic fractional neutral system with multiple time varying delay in control of the form

$$\begin{aligned} {}^C D^\alpha \Big(x(t) - g(t) \Big) &= Ax(t) + \sum_{i=0}^{M} B_i u(h_i(t)) + f(t) \\ &\quad + \sigma(t) \frac{\mathrm{d}W(t)}{\mathrm{d}t}, t \in J, \\ x(0) &= x_0 \in \mathcal{H}_2^0, \end{aligned} \tag{1}$$

where $0 < \alpha \leq 1$, $\alpha \neq \frac{1}{2}$, A: $X \to X$ is a bounded linear operator, $W(t)$ is a K-valued Wiener process with positive symmetric trace class covariance operator, $\sigma : J \to \mathbb{L}_2^0(K, X)$ (where $\mathbb{L}_2^0$ is the space of Hilbert–Schmidt operators [11]), f, g: $J \to X$ are continuous functions and g is continuously differentiable, $u \in U_{ad}$, a Hilbert space of admissible control functions and for $i = 0, 1, 2, \ldots, M$, B_i: $U \to X$ are bounded linear operators. The function $h_i : J \to \mathbb{R}$, $i = 0, 1, 2, \ldots, M$ are twice continuously differentiable and strictly increasing in J. Moreover

$h_i(t) \leq t$ for $t \in J, i = 0, 1, 2, \ldots, M$.

Let us introduce the time lead functions

$r_i(t) : [h_i(0), h_i(T)] \to J, i = 0, 1, 2, \ldots, M$

such that $r_i(h_i(t)) = t$ for $t \in J$. We further assume $h_0(t) = t$ and for $t = T$ the function $h_i(t)$ satisfy the inequalities

$$\begin{aligned} &h_M(T) \leq h_{M-1}(T) \leq \cdots \leq h_{m+1}(T) \leq 0 \\ &\quad = h_m(T) < h_{m-1}(T) = \cdots = h_1(T) = h_0(T) \end{aligned} \tag{2}$$

The usual definition of complete state is assumed.

Definition 2.3 [4]**.** The pair $y(t) = \{x(t), u_t\}$ where u_t denotes the function on $[\min_i h_i(t), t)$, defined by $u_t(s) = u(s)$ is said to be the complete state of system (1) at time t.

We obtain the solution of the system (1) as in [5] by using the following lemma and hypothesis.

Lemma 2.1 *[20]. Suppose that A is a linear bounded operator defined on a Banach space, and assume that $\|A\| < 1$. Then $(I-A)^{-1}$ is linear, bounded and*

$$(I-A)^{-1} = \sum_{k=0}^{\infty} A^k.$$

The convergence of the above series is in the operator norm and $\|(I-A)^{-1}\| \leq (1-\|A\|)^{-1}$.

Let us assume the following hypothesis:

(H1) The operator $A \in \mathbb{L}(X)$ and $\|A\|^2 < \frac{(2\alpha-1)(\Gamma(\alpha))^2}{T^{2\alpha}}$.

Let $x \in \mathcal{H}_2$, then by (H1), we have

$$\begin{aligned} \|(I^\alpha A)x\|_{\mathcal{H}_2} &\leq \frac{T}{(\Gamma(\alpha))^2} \sup_{t\in J} \int_0^t (t-s)^{2\alpha-2} \mathbf{E}\|Ax(s)\|_X^2 \mathrm{d}s \\ &\leq \frac{T^{2\alpha}}{(2\alpha-1)(\Gamma(\alpha))^2} \sup_{t\in J} \mathbf{E}\|Ax\|_X^2 \leq \|x\|_{\mathcal{H}_2}, \end{aligned}$$

which implies that $\|I^\alpha A\| < 1$. Hence by Lemma 2.1 we conclude that $(I - I^\alpha A)^{-1}$ is a bounded linear operator satisfying $(I - I^\alpha A)^{-1} = \sum_{k=0}^{\infty} (I^\alpha A)^k$ and $\|(I - I^\alpha A)^{-1}\| \leq \frac{1}{1-\|I^\alpha A\|}$. On the other hand, taking I^α on both sides of (1) and using Lemma 2.1 we obtain

$$\begin{aligned} x(t) &= E_\alpha(At^\alpha)[x_0 - g(0)] + g(t) \\ &\quad + \int_0^t A(t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha) g(s) \mathrm{d}s \\ &\quad + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha) f(s) \mathrm{d}s \end{aligned}$$

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