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Complex vague set based concept lattice

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ABSTRACT

Recently, the calculus of concept lattice is extended from unipolar to bipolar fuzzy space for precise measurement of vagueness in the attributes based on their acceptation and rejection part. These extensions still unable to highlight the uncertainty in vague attributes and measurement of fluctuation at given phase of time. To conquer this problem, current paper proposed a method for adequate analysis of vagueness and uncertainty in data with fuzzy attributes using the amplitude and phase term of a defined complex vague set based concept lattice. In addition, the analysis derived from the proposed method is compared with CVSS method through an illustrative example.

1. Introduction

Recent years, much attention has been paid towards precise analysis of uncertainty and vagueness in the given data set with fuzzy attributes. The uncertainty and vagueness in data coexists simultaneously at the given phase of time. One of the suitable example is large number of data generated from medical diagnoses may contain lots of incomplete, uncertain and vague information. Handling these type of complex or dynamic data set is a major issue for the researcher communities. To crush this problem, recently Ye [1] tried to characterize the uncertainty and vagueness in medical diagnoses data set based on its truth, indeterminacy and falsity membership–value. However uncertainty and vagueness in the symptoms of medical diagnoses data set changes at each interval of time [2]. To represent these types of dynamic data set recently properties of complex vague set [3] is introduced using the extensive properties of complex fuzzy logic [4–6]. The current paper put forward effort to discover all the hidden pattern in a given complex data set. To elaborate the proposed method current paper focuses on medical diagnoses data set and its hidden pattern using the properties of Formal Concept Analysis (FCA). The calculus of FCA is already applied in analysis of gene expression data [2], Chinese medicine data [7], Breast cancer data [8], Health care data [9] and TB data [10]. All of these available approaches focused on finding pattern in medical diagnoses data in binary attributes. These methods lacks in handling the data set beyond the binary attributes and their fluctuation at given phase of time. The reason is to measure the vagueness and fluctuation in the uncertainty calculus of complex vague set, complex vague lattice and complex vague graph is required which is at infancy stage. To fill this backdrop, the current paper aimed at depth analysis of complex fuzzy set, its partial ordering visualization in the concept lattice using the extensive properties of FCA.

FCA is one of the well–established mathematical model for data analysis and processing, based on applied abstract algebra [11]. The calculus of FCA provides a an alternative way to discover all the hidden pattern (i.e. formal concept) in a given data set. The generated formal concepts are nothing but a pair of objects (i.e. extent) and their common attributes (i.e. intent) which are closed with Galois connection. It can be considered as a basic unit of thought for knowledge processing tasks [12]. All of the generated formal concepts can be displayed in compact arrangement of their generalization and specialization properties of a given concept lattice. This hierarchical order visualization provides an adequate way to refine the interested pattern in the given data set when compared to its numerical representation. To intensify the knowledge processing tasks, the calculus of FCA expedite with fuzzy [13], interval [14–16], bipolar [17], three–polar [18], possibility [19], rough set [20] and other extensive theory [21–23]. For defining the vagueness in attributes through unipolar [0,1], bipolar$[0, 1]^2$ or three–polar $[0,1]^3$ fuzzy space based on their acceptation and rejection part. However, these available approaches are unable to highlight the fluctuation in uncertainty and vagueness at given phase of time [23]. In general the uncertainty in data occurs at each interval of time whereas vagueness is created due to problem in computational linguistics (like tall, young or bald). The uncertainty may be derived from the factors like inconsistency, incompleteness, or

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Table 1
Inevitable appearance of vagueness in medical data set in form of context.

Conditions	Objects	Attributes	Relation
a	Complete	Complete	Vague (or) Complex
b	Incomplete or Vague	Complete	Vague (or) Complex
c	Complete	Incomplete or Vague	Vague (or) Complex
d	Incomplete or Vague	Incomplete or Vague	Vague (or) Complex

Table 2
Comparison among interval, bipolar, vague and complex vague set.

	Interval	Bipolar	Vague	Complex vague set
Domain	Universe of discourse	Universe of discourse	Universe of discourse	
Co–domain	Unipolar	Bipolar interval- [0,1]	[0,1] [-1,0) × (0, 1]	
Uncertainty	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes
Falsity	No	No	No	Yes
Positive	[0,1]	[0,1]	(0, 1]	[0,1]
Negative	No	No	[−1, 0)	[0,1]
Sharp boundaries	Yes	Yes	No	No
Unit circle	[0,1]	[0,1]	[0,2]	[0,1]
Amplitude term	Yes	Yes	Yes	Yes
Phase term	No	No	No	Yes

seasonality. It occurs when, the data shows cyclic or periodic pattern in a given phase of time. Temperature is one of the suitable example to define the uncertainty and its fluctuation at given phase of time (i.e. year). The temperature 22 ° is considered as cool in the summer whereas the same temperature 22 ° is considered as warm in the winter season. This type of inconsistency and fluctuation in the given attributes creates uncertainty in the data. The vagueness is somehow related with the fuzziness available in given attributes (i.e. like young, tall or bald). It can be measured based on evidence to support (i.e. true membership –value) or reject (i.e. false membership–value) the attributes for the given context. It means vagueness occurs when a fuzzy attribute cannot be defined via a sharp boundary. This situation generally can be found in medical diagnoses data set, stock market and time series data. As for example, How much hair loss is required to consider a person is bald or not ?, How much loss of vision is required to consider a person is legally blind or not?, What is the point of conception from date of birth to declare a person as human being? These all fuzzy attributes contain vagueness which cannot be defined by a precise boundaries as shown in Table 1. To represent these types of attributes, an expert requires some evidence to support or reject them in a seized scale [0, 1]. For this purpose, recently properties of vague set [24], vague graph [25,26], vague hypergraph [27,28], vague soft set [29], and vague lattice [30–32] is studied to expedite its applications [33]. Subsequently, for measurement of uncertainty and its fluctuation in the attributes [34] properties of complex vague set [3] is introduced using the calculus of complex fuzzy set [4], complex fuzzy logic [5,6] and vague soft set [29]. Motivated from these recent studies current paper focuses on exploring the calculus of complex vague set with concept lattice for handling complex data set. To fulfill this objective interval–valued, bipolar, vague and complex fuzzy set is comparatively studied in Table 2. This table shows that the complex vague set provides more precise representation of vagueness and uncertainty in the fuzzy attributes using the amplitude and phase term.

To achieve the goal, a method is proposed, in this paper to generate all the hidden pattern (i.e. formal concepts) in a given complex vague context using the amplitude and phase of a defined complex vague set [3], vague graph [25,26], and concept lattice [11,12,35–41]. To explore the properties of complex vague relation [41–43] for refining the knowledge processing tasks using technique of concept lattice [23,44,45]. To fulfill this backdrop, the proposed method is applied on a medical diagnoses data set with step by step illustration. The motivation is to improve the medical diagnoses data processing tasks using a mathematical model rather than traditional methods as it affects human life directly. The objective is to provide an accurate result for the adequate analysis of disease and its recovery for a given phase of time. To validate the results, analysis derived from the proposed method is compared with CVSS method [3] with an illustrative example.

Rest of the paper is organized as follows: Section 2 provides a brief background about FCA with the vague setting. Section 3 contains the proposed method for generating the complex vague concepts and its illustration in Section 4. Section 5 provide discussions followed by conclusions, and references.

2. Formal concept analysis with the vague setting

There are many data set like (http://indianalgae.co.in/) which contains vague attributes [34]. Medical data set is one of the suitable example which contains lots of incomplete, inconsistent and vague information. To represent these type of attributes an expert need evidence to accept or reject them in a seized scale [0, 1]. To fill this backdrop, Gau and Buehrer [24] introduced properties of vague set. In this section some basic preliminaries about FCA with vague setting is given for handling the data with vague attributes.

Definition 1. (Formal fuzzy context) [13]: A formal fuzzy context $\mathbf{F} = (X, Y, \tilde{R})$ is a fuzzy matrix having X as set of objects, Y as set of attributes, and L–relation among them i.e. $\tilde{R}$: $X \times Y \rightarrow L$. In general the relation $\tilde{R}$ represents non–zero fuzzy membership value at which the object $x \in X$ has the attribute $y \in Y$ in [0, 1] where L is a support set of some complete residuated lattice $\mathbf{L}$ [13].

Definition 2. (Formal vague context) [24,32]: A formal vague context $\mathbf{F} = (X, Y, \tilde{R})$ represents set of objects (X), set of vague attributes (Y) and a vague relation $\tilde{R}$ between them $\tilde{R} = \{\langle (x, y), t_{\tilde{R}}(x, y), f_{\tilde{R}}(x, y)\rangle | x \in X, y \in Y\}$. As for example, a patient suffer from pneumonia or not can be represented through an evidence for its acceptation (i.e. positive) and rejection (i.e. false membership) value of a defined vague set and vague relation among them.

Definition 3. (Residuated lattice) [35] : It is a basic structure of truth degrees $\mathbf{L} = (L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ in which 1 represents greatest elements and 0 represents least elements respectively. $\mathbf{L}$ is a complete residuated lattice iff :

(1) $(L, \wedge, \vee, 0, 1)$ is a complete lattice.
(2) $(L, \otimes, 1)$ is commutative monoid.
(3) $\otimes$ and $\rightarrow$ are adjoint operators called as multiplication and residuum, respectively i.e. $a \otimes b \leq c$ iff $a \leq b \rightarrow c, \forall a, b, c \in L$.

The operators $\otimes$ and $\rightarrow$ are defined distinctly by Lukasiewicz, G ödel, and Goguen t–norms and their residua as given below [36];

Lukasiewicz:

- $a \otimes b = \max(a + b - 1, 0)$,
- $a \rightarrow b = \min(1 - a + b, 1)$.

G ödel:

- $a \otimes b = \min(a, b)$,
- $a \rightarrow b = 1$ if $a \leq b$, otherwise b.