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Explicit formula for the valuation of catastrophe put option with exponential jump and default risk

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ABSTRACT

This paper concerns a catastrophe put option with default risk. Catastrophe events are described by the exponential jump model, and the default event of the option issuer is specified by the intensity based model with a stochastic intensity. Under this model, we derive the explicit analytical pricing formula of a catastrophe put option with default risk by using the multidimensional Girsanov theorem repeatedly. We also observe the effects of default risk on the prices of a catastrophe put option through the numerical experiment.

1. Introduction

In finance, the default risk (or credit risk) has classically been dealt with two types of approaches: the intensity based approach (or reduced-form) and the structural approach (or firm value-based). The intensity-based models have been developed with assumption that default is controlled by the first jump of a given counting process with intensity. Namely, the bankruptcy in this model is triggered by the first jump of a Poisson process and the default time of underlying asset depends on some intensity process. On the other hands, the structural models depend on the dynamic of the firm's value. The structural model for default risk was first proposed by Merton [1]. The Merton model assumes that firm's value is defined by the value of debt and the value of equity. Defaults of firms under the Merton model just occur at the maturity of bond if the debt holders can not redeem their duty. Recently, based on the structural of Merton, many researchers have studied the pricing of options with default risk, which called Vulnerable option.

Johnson and Stulz [2] first consider the default risk for option pricing by assuming that the options depend on the liabilities of the option issuers. Based on the structural model of Merton, if the value of the option is less than the value of the option issuer at the maturity, the default of the option issuer happens and the option investor takes the assets of the option issuer. For a more realistic environment, Klein [3] extended the result of Johnson and Stulz by allowing not only the correlation between the option issuers asset and the underlying asset but also the proportional recovery of nominal claims when the default happens. Klein and Inglis [4] provide the pricing formula of vulnerable options with the stochastic interest rate using the partial differential equation method. Chang and Hung [5] derived analytic pricing formula for the vulnerable American options under the Black-Scholes model. Kim and Koo [6] used the Mellin transform approach to solve the partial differential equation of Exchange option with credit risk under the model of Klein [3]. In addition, in recent years, many researchers have studied the vulnerable options for the real financial market, which follows the stochastic volatility model [7–10]. Concretely, Yang et al. [11] and Lee et al. [12]. considered the vulnerable option when the volatilities of underlying assets follow the stochastic dynamics. Kim [13] also adopted a regime-switching model to consider the two underlying assets affected by a stochastically changing market environment. These studies dealt with the valuations of options with default risk based on the structural model. On the other hand, Fard [14] provided recently the analytical pricing formulas of European vulnerable options based on an intensity based model. Concretely, Fard [14] studied the vulnerable options when the underlying asset process follows a generalized jump-diffusion model.

The insurance losses from the global natural catastrophe events such as floods, earthquakes, typhoons and forest fires have grown rapidly in recent years. The increase of insurance losses has imposed the insurance companies to find the ways to hedge against risks caused by the catastrophe events. In order to hedge the risks,

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insurance companies have used the financial derivatives such as bonds, futures and options.

Catastrophe put option is one of the most popular financial derivatives for catastrophe risk management. Catastrophe put options first were considered by Cox et al. [15]. The pricing model of them assumes the underlying asset process follows the geometric Brownian motion with downward jump, and the catastrophe event influences only the stock price. Janimunggal and Wang [16] develop the results of Cox et al. [15] by providing the pricing formula of the European Catastrophe put option with stochastic interest rate and compound Poisson processes. Chang and Hung [17] provided the explicit analytic pricing formulas for catastrophe put options when the underlying asset process follows a Lévy process with finite activity. Lin and Wang [18] investigated the perpetual American catastrophe put options by using a penalty function approach. In addition, Yu [19] proposed the catastrophe put option pricing model by assuming that the underlying asset processes follow an exponential jump process with jump terms modeled by two compound Poisson processes. Wang [20] considered a new class of catastrophe put options. More concretely, Wang [20] studied catastrophe put options with target variance, which represents the expectation of the insurance company for the future realized variance.

Recently, the pricing models for catastrophe put option with default risk have been studied. Jiang et al. [21] first proposed the valuation model of catastrophe put option by assuming of default risk model proposed by Klein [3]. Wang [22] developed the pricing model of catastrophe put options with default risk when default of option issuer allows to occur at any time prior to maturity of option. Here, Wang [22] assumed that catastrophe event follows a doubly stochastic Poisson process, and the stock process is affected by the catastrophe losses. All of them utilized a structural model for modeling of the default risk. In this paper, we consider the intensity based model for pricing of the catastrophe put option with default risk based on the model of Fard [14].

This paper is organized as follows. In Section 2, we introduce the model for the underlying asset, the stochastic interest rate, and the catastrophe losses. In Section 3, we derive the explicit pricing formula for the catastrophe put option with default risk. In Section 4, we provide some examples of option prices to observe the effects of default risk with respect to model parameters. Finally, Section 5 provides the concluding remarks.

2. The model

We assume that a given filtered complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}(t)\}, P)$ satisfies the usual conditions, where P presents a risk-neutral probability measure and the filtration $\{\mathcal{F}(t)\}$. Under the measure P, as in Kou and Wang [23], the underlying asset price $S(t)$ with the exponential jump diffusion model[1] is given by

$$S(t) = S(0)\exp\left\{\int_0^t r(s)ds - \left(\frac{1}{2}\sigma_1^2 + \lambda_N\zeta\right)t + \sigma_1\, dW_1(t) - c\sum_{i=1}^{N(t)} l_i\right\}, \quad (1)$$

where σ_1 is a volatility of underlying asset, c is a positive conversion factor, $W_1(t)$ is a standard Brownian motion and $N(t)$ is a Poisson process with intensity λ_N. We also assume that the risk-neutral instantaneous shot rate $r(t)$ is governed by the Vasicek model [26] as

$$dr(t) = k(\theta - r(t))dt + \sigma_2 dW_2(t),$$

where k, θ and σ_2 are constants and $W_2(t)$ is a standard Brownian motion under the measure P satisfying $dW_1(t)dW_2(t) = \rho_{12}dt$. The log jump sizes (the sizes of loss) $\{l_1, l_2, \cdots\}$ form a sequence of independent identically distributed random variables with the double exponential density $f_l(x)$ defined by

$$f_l(x) = p\eta_1 e^{-\eta_1 x}\mathbf{1}_{\{x\geq 0\}} + q\eta_2 e^{\eta_2 x}\mathbf{1}_{\{x<0\}},$$

where $p,\ q \geq 0$, $p+q=1$, $\eta_1 > 0$, $\eta_2 > 0$, and

$$\zeta = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1.$$

Moreover, all sources of randomness $W_1(t)$, $N(t)$ and $\{l_1, l_2, \cdots\}$ are independent under the measure P. Since all jumps of the underlying asset by catastrophic events are downward jumps, as in Chang and Hung [17], we consider only the negative exponentially distributed jump in the exponential jump diffusion model. Then the underlying asset price $S(t)$ under the risk-neutral measure P is given by

$$S(t) = S(0)\exp\left\{\int_0^t r(s)ds - \left(\frac{1}{2}\sigma_1^2 + \lambda^*\zeta^*\right)t + \sigma_1 W_1(t) - c\sum_{i=1}^{N^*(t)} l_i^d\right\}, \quad (2)$$

where $N^*(t)$ is the Poisson process with intensity $\lambda^* = \lambda_N(\zeta+1)$ and $\zeta^* = \frac{\eta_2^*}{\eta_2^*+1} - 1$ with $\eta_2^* = \eta_2 + 1$ and $\zeta = \frac{\eta_2}{\eta_2+1} - 1$. In addition, the log jump sizes $\{-l_1^d, -l_2^d, \cdots\}$ are independent identically distributed random variables indicating ith downward jump with density function defined by

$$\nu(x) = \lambda^* f^d(x) = \lambda^*\eta_2^* e^{\eta_2^* x}\mathbf{1}_{\{x<0\}},\ \eta_2^* > 0.$$

If we define the loss process of the insured to be $L(t) = \sum_{i=1}^{N^*(t)} l_i^d$, the payoff of the catastrophe put option is defined as

$$\text{payoff} = \mathbf{1}_{\{L(T)-L(0)>\bar{L}\}}\begin{cases} K - S(T), & S(T) < K \text{ and } L(T) - L(0) > \bar{L}, \\ 0, & L(T) - L(0) \leq \bar{L}, \end{cases}$$

where T is the maturity, $\bar{L}$ is the trigger level of losses, K is the strike price at which the issuer is obligate to purchase unit shares if the cumulative losses go over the level $\bar{L}$, and $L(T) - L(0)$ is the total losses of the insured over the time $[0, T)$. With these assumptions, we consider the catastrophe put option with default risk under the intensity based model in the next section.

3. Pricing catastrophe put option with default risk

We consider the valuation of catastrophe put option with default risk based on an intensity based model in this section. As in Fard [14], under the risk-neutral measure P, we assume that the default intensity process is controlled by the following process

$$d\lambda(t) = a(b - \lambda(t))dt + \sigma_3 dW_3(t),$$

where σ_3 is a positive constant, and $W_3(t)$ is a standard Brownian motion satisfying $dW_1(t)dW_3(t) = \rho_{13}dt$, $dW_2(t)dW_3(t) = \rho_{23}dt$ with the Brownian motions $W_1(t)$ and $W_2(t)$ defined in Section 2. We also define the filtration $\mathcal{F}(t)$ generated by $\mathcal{F}(t) = \mathcal{F}^S(t) \vee \mathcal{F}^r(t) \vee \mathcal{F}^\lambda(t) \vee \mathcal{H}(t)$, where $\mathcal{F}^S = \sigma(S(t), s \leq t), \mathcal{F}^r = \sigma(r(t), s \leq t), \mathcal{F}^\lambda = \sigma(\lambda(t), s \leq t)$ and $\mathcal{H}(t) = \sigma(\mathbf{1}_{\{\tau\leq t\}}, s \leq t)$. Then the value V of the catastrophe put option with default risk at time 0 is given by

$$V = \mathbb{E}[e^{-\int_0^T r(s)ds}(w(K - S(T))^+\mathbf{1}_{\{\tau\leq T, L(T)-L(0)\geq\bar{L}\}} + (K - S(T))^+\mathbf{1}_{\{\tau\geq T, L(T)-L(0)\geq\bar{L}\}})|\mathcal{F}(0)], \quad (3)$$

[1] The exponential jump model of Kou and Wang is one of the most popular jump models for valuing the financial derivatives. Many researchers have developed the exponential jump model, and have adopted the model to price various financial derivatives (e.g., see [24,25]).