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Modulation instability of broad optical beams in unbiased photorefractive pyroelectric crystals

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ABSTRACT

We present a study of the one dimensional modulation instability due to a broad optical beam in pyroelectric photorefractive crystals where the space charge field is formed due to solely the pyrolectric effect. The one-dimensional growth rate of the modulation instability depends upon the intensity of the incident beam and the magnitude of the temperature change. Relevant example of a Strontium Barium Niobate crystal is taken to illustrate the theoretical analysis.

1. Introduction

Photorefractive solitons have attracted much attention in the past two decades due to their applications in optical switching and routing. They have been observed as screening photorefractive solitons [1], screening photovoltaic solitons [2], photovoltaic solitons [3] and in centrosymmetric photorefractive media. Screening solitons require an external electric field which screens the space charge field. In photovoltaic crystals, the bulk photovoltaic field is responsible for the formation of the space charge field. There have been extensive studies on various characteristics of solitons in photorefractive media [4–15]. Recently, there have been studies on soliton formation in pyroelectric photorefractive media [16–18]. In [17], the authors have predicted the existence of solitons due to solely the pyroelectric effect in SBN crystals. We have, recently studied the incoherently coupled soliton pairs in such pyroelectric photorefractive media [19]. In a ferroelectric crystal, the net electric field inside the crystal is zero. This is because the charge distribution on the crystal faces compensates the field due to spontaneous polarization. A temperature change causes a spontaneous polarization change and hence, a transient electric field E_{py}. This is called as the pyroelectric field. This field is not compensated immediately and consequently, a drift current can be set up mimicking the effect of an external electric field applied to the crystal. Now, this field is locally screened due to the space charge field formed due to the photorefractive effect and hence, a self trapped beam results. We emphasize that as compared to the formation of screening solitons, replacing the external electric field with the temperature change induced pyroelectric field for soliton formation has clear advantages. Firstly, we need not know the direction of the crystal c-axis since the pyroelectric field is always along the *c*-axis, i.e., in one direction for heating and in the reverse direction for cooling. Secondly, no electrodes are required on the crystal [20,21].

Modulation instability is a common phenomenon which is inherent in many non-linear systems [22–26]. It occurs in the same parameter space in which solitons are observed and is considered as a precursor to soliton formation. It manifests as a growth of spatial frequency sidebands in a broad optical beam. There have been investigations of the modulation instability of screening solitons and screening photovoltaic solitons [27–29], photorefractive solitons in centrosymmetric media [30]. The modulation instability in unbiased photorefractive crystals in which the space charge field is formed due to the pyroelectric effect alone has not been studied as yet. Hence, it is of interest to study the modulation instability of such photorefractive solitons which form due to solely the pyroelectric effects. We shall study this in detail in the current paper taking a relevant example of an SBN crystal.

2. Theory

Let us consider a broad optical beam propagating in a photorefractive crystal having finite pyroelectric coefficient. The beam is allowed to diffract only along the *x*-direction and propagates along the *z*-direction. The crystal is kept in contact with a metallic plate which has its temperature accurately controlled by an external agency. The crystal is duly covered by an insulating plastic cover. In our analysis, we shall take the photorefractive pyroelectric crystal to be Strontium Barium Niobate, $Sr_{0.6}Ba_{0.4}Nb_2O_6$ which

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is abbreviated as SBN henceforth. The incident beam is expressed as a slowly varying envelope $\vec{E} = \hat{x}A(x,z)\exp(ikz)$ where $k = k_0 n_e$, $k_0 = \frac{2\pi}{\lambda_0}$. Here, n_e is the unperturbed extraordinary refractive index and λ_0 is the free space wavelength. Under these conditions, the evolution of the optical beams is given by [1],

$$\left(i\frac{\partial}{\partial z} + \frac{1}{2k}\frac{\partial^2}{\partial x^2} + \frac{k}{n_e}\Delta n\right)A(x,z) = 0 \quad (1)$$

$$\Delta n = -\frac{1}{2}n_e^3 r_{eff} E_{sc} \quad (2)$$

where r_{eff} is the electro-optic coefficient and $E_{sc} = E_{pysc}$ is the space charge field in the medium resulting from the transient pyroelectric field and is [17],

$$E_{pysc} = -E_{py}\frac{I}{I+I_d} \quad (3)$$

E_{py} is the transient pyroelectric field and is given by [18],

$$E_{py} = -\frac{1}{\varepsilon_0\varepsilon_r}\frac{\partial P}{\partial T}\Delta T \quad (4)$$

where $\frac{\partial P}{\partial T}$ is the pyroelectric coefficient and ΔT is the magnitude of the temperature change of the crystal and I_{d} is the dark irradiance, i.e, the intensity at constant illumination region of the crystal, $I(x \to \infty)$

From the expression for the space charge field (3), we can infer that the value and sign of the space charge field depends upon E_{py} which, in turn can be varied by controlling the change in temperature, i.e, by the requisite heating, or cooling effects. The total optical power density for the two mutually incoherent beams can be obtained by the Poynting flux,

$$I = \frac{n_e}{2\eta_0}\left(|A|^2\right) \quad (5)$$

with $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$.

Substituting E_{pysc} and Δn in (1), one gets the following equation,

$$iU_z + \frac{1}{2}U_{xx} + \beta E_{py}\frac{|U|^2}{1+|U|^2}U = 0 \quad (6)$$

where we have written,

$$A = \left(\frac{2\eta_0 I_d}{n_e}\right)^{1/2}U,\ U_z = \frac{\partial U}{\partial z},\ U_{xx} = \frac{\partial^2 U}{\partial x^2},\ I = I_d|U|^2,$$
$$\beta = \frac{1}{2}\left(k_0 n_e^3 r_{eff}\right)$$

The plane wave broad beam solution of (6) takes the form,

$$U = r^{1/2}\exp\left[i\beta E_{py}\left\{\frac{r}{1+r}\right\}z\right] \quad (7)$$

For studying the modulation instability of the plane wave, we express,

$$U = [r^{1/2} + \sigma(x,z)]\exp\left[i\beta E_{py}\left\{\frac{r}{1+r}\right\}z\right] \quad (8)$$

where $\sigma(x, z)$ contains two sideband plane waves and is the weak modulation term added to the steady state solution (7). This weak perturbation satisfies

$$|\sigma(x,z)| << r^{1/2} \quad (9)$$

and is assumed,

$$\sigma = a(z)\exp(ipx) + b(z)\exp(-ipx) \quad (10)$$

We want to investigate the possible exponential growth of this perturbation. For this purpose, substituting (8) in (6) and using the linearizing condition (9), we get the following evolution equation for the perturbation $\sigma(x, z)$,

$$i\frac{\partial\sigma}{\partial z} + \frac{1}{2k}\frac{\partial^2\sigma}{\partial x^2} + \beta E_{py}\frac{r}{(1+r)^2}(\sigma + \sigma^*) = 0 \quad (11)$$

Substituting (10) in (11), we obtain the following coupled equations,

$$i\frac{da}{dz} - \frac{1}{2k}p^2 a + \beta E_{py}\frac{r}{(1+r)^2}(a + b^*) = 0 \quad (12)$$

$$i\frac{db}{dz} - \frac{1}{2k}p^2 b + \beta E_{py}\frac{r}{(1+r)^2}(a^* + b) = 0 \quad (13)$$

By decoupling Eqs. (12) and (13), we get simply,

$$\frac{d^2a}{dz^2} = \left[\beta E_{py}\frac{r}{(1+r)^2}\frac{p^2}{k} - \frac{p^4}{4k^2}\right]a \quad (14)$$

$$\frac{d^2b}{dz^2} = \left[\beta E_{py}\frac{r}{(1+r)^2}\frac{p^2}{k} - \frac{p^4}{4k^2}\right]b \quad (15)$$

Eqns. (14) and (15) are of the form of a linear second order differential equation. Hence, their solutions are of the form $\exp(\omega z)$ with

$$\omega = \left[\beta E_{py}\frac{r}{(1+r)^2}\frac{p^2}{k} - \frac{p^4}{4k^2}\right]^{1/2} \quad (16)$$

Hence, the local modulation instability gain is given as,

$$g = \mathrm{Re}\left\{\left[\beta E_{py}\frac{r}{(1+r)^2}\frac{p^2}{k} - \frac{p^4}{4k^2}\right]^{1/2}\right\} \quad (17)$$

From (17), we can find the maximum modulation instability gain as,

$$g_{\max} = \left[\frac{1}{2}k_0 E_{py} n_e^3 r_{eff}\frac{r}{(1+r)^2}\right] \quad (18)$$

and the associated spatial frequency is,

$$p_{max} = \frac{k_0 n_e^2}{1+r}\left[r_{eff}E_{py}r\right]^{1/2} \quad (19)$$

3. Results and Discussion

We consider the SBN crystal and hence, we take the following parameters [17,31,32], $n_e = 2.35$, $\lambda_0 = 532$ nm., $r_{eff} = 237 \times 10^{-12}$ m/V, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\varepsilon_{\mathrm{r}} = 3400$, $\frac{\partial P}{\partial T} = -3 \times 10^{-4}$ Cm^{-2}K^{-1}, $r = 10$.

For this set of values, we get, $\beta = 0.018$ and $k = 2.774 \times 10^7$. Fig. 1 shows the MI gain g as a function of p/k at $\Delta T = 20$ °C. p/k is the angle at which the plane wave components of the $\sigma(x, z)$ perturbation propagate with respect to the broad optical beam. We note that the MI gain g first increases with an increase in p/k, and then after reaching the peak value, g decreases and finally becomes zero after a certain value of p/k.

Now, it is important to note that the MI gain g depends upon E_{py} and hence ΔT which is the change in temperature of the photorefractive pyroelectric crystal. Hence, it is interesting to study the MI gain g with varying ΔT and p/k. Fig. 2 shows the dependence of g simultaneously with ΔT and p/k for $r = 1$. We can see that as the value of ΔT increases, the peak MI gain g_{max} increases. Also, as the value of ΔT increases, the value of p/k after which g becomes zero also increases, i.e., the range of values of p/k for which there is a finite MI gain increases with an increase in ΔT.