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Razumikhin method for impulsive functional differential equations of neutral type[☆]

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ABSTRACT

Although the well-known Razumikhin method has been well developed for the stability of functional differential equations with or without impulses and it is very useful in applications, so far there is almost no result of Razumikhin type on stability of *impulsive functional differential equations of neutral type*. The purpose of this paper is to close this gap and establish some Razumikhin-based stability results for impulsive functional differential equations of neutral type. A kind of auxiliary function $N(t)$ that has great randomicity is introduced to Razumikhin condition. Some examples are given to show the effectiveness and advantages of the developed method.

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1. Introduction

Qualitative properties of the mathematical theory of impulsive differential equations have been developed by a large number of researchers in the past several years, see [1–9]. Such kind of equations represents a natural framework for mathematical modelling of several processes and phenomena studied in biology, Engineering, physics technology and so on [10–14]. One of the important applications of impulsive differential equations to real problems is to find efficient stability conditions for their solutions. Now there have been many interesting methods and results on stability analysis of various impulsive differential equations, especially for impulsive differential equations involving delay effects which are called impulsive functional differential equations (IFDEs) [15–18].

Razumikhin method was initially proposed by Razumikhin [19,20] for the ordinary differential delay equations and was then developed by several researchers to more general functional differential equations [21–26], especially for IFDEs [27–33]. The idea and advantage of Razumikhin method coupled with Lyapunov functions for IFDEs is to consider the effects of delay and impulse via the relationship between the current state and the history state of Lyapunov function, which could not only avoid the construction of the complex Lyapunov functional, but also show the effects of impulsive perturbations very well. In many cases Lyapunov–Razumikhin method can give an efficient way to deal with the dynamics of delayed system which cannot be solved via Lyapunov functional methods. For example, it is feasible to study the impulsive stabilization problem of IFDEs by employing Lyapunov–Razumikhin method, but in such case it is difficult to utilize the method of Lyapunov functional due to the existence of impulsive control, see [31–33]. Based on Lyapunov–Razumikhin method, Li et al. [34,35] designed the impulsive controller and studied the synchronization control problem of delayed chaotic systems, but the Lyapunov functional method is hard to apply from the impulsive control point of view. Although there are so many advantages, to our knowledge, there is almost no result, based on Razumikhin method, reported for the stability theory of IFDEs of neutral type. As mentioned in [24,36], there are a number of difficulties that one must face in developing the corresponding stability theory of functional differential equations of neutral type by using Razumikhin–

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Lyapunov function method, not to mention the case for IFDEs. For example, the general ideas constructing Lyapunov functions are invalid for IFDEs of neutral type due to the existence of neutral term and impulses. Moreover, it is nature that Razumikhin condition should include some information of the neutral term since there exist some time delays in neutral term of IFDEs of neutral type, which is hard to handle in the operation.

In recent years, some results on the existence, controllability and stability of solutions for IFDEs of neutral type have been presented in [37–42]. In [40], the asymptotic behavior of nonlinear neutral delay differential equations with impulses were considered by establishing proper Lyapunov functions and some analysis techniques. However, the impulses considered were just some special cases. In [41], some results ensuring global exponential stability of IFDEs of neutral type were derived via impulsive delay inequality and some analysis techniques, which were very popular in the application of dynamical analysis of neural networks. However, the results were based on M-matrix theory and can only be applied to some special autonomous systems. Hence, new methods and technique for stability analysis of IFDEs of neutral type should be explored and developed.

Our goal in this paper is to develop the Razumikhin method to IFDEs of neutral type and establish some Razumikhin theorems. We introduce a kind of auxiliary function $N(t)$ that has great randomicity in Razumikhin condition. Some Razumikhin-type theorems on uniform stability and uniform asymptotic stability are established, which are seldom reported in the literature. Furthermore, even there is no impulse, the developed results in this paper are more general than some existing results [43–46]. The work is organized as follows. In Section 2, we introduce some basic definitions and notations. In Section 3, we present the main results. In Section 4, some examples are given to show the powerfulness of our new results. Finally, the paper is concluded in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of positive real numbers and $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ the n-dimensional and $n \times m$-dimensional real spaces equipped with the Euclidean norm $\|\cdot\|$. Let $\mathbb{Z}_+$ denote the set of positive integers, i.e., $\mathbb{Z}_+ = \{1, 2, \ldots\}$. $\mathbb{B} = \{\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and satisfies $\varphi(s) \geq s$ for $s > 0\}$. $\Lambda = \{1, 2, \ldots, n\}$. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k (1 \leq k \leq n)$, $C(J, S) = \{\varphi : J \to S$ is continuous$\}$, $PC(J, S) = \{\varphi : J \to S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$ and $PC^1(J, S) = \{\varphi : J \to S$ is continuously differentiable everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$, $\dot\varphi(t^+)$ and $\dot\varphi(t^-)$ exist, $\varphi(t^+) = \varphi(t)$, $\dot\varphi(t^+) = \dot\varphi(t)$, where $\dot\varphi$ denotes the derivative of $\varphi\}$. The impulse times t_k satisfy $0 \leq t_0 < t_1 < \ldots < t_k <\to +\infty$ as $k \to \infty$.

Consider the following IFDEs of neutral type:

$$\begin{cases} \dot x(t) = f(t, x_t, \dot x(t-\tau(t))), & t \in [t_{k-1}, t_k), \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(t_k, x(t_k^-)), & k \in \mathbb{Z}_+, \end{cases} \tag{1}$$

where $0 < \tau \leq \tau(t) \leq r$, and τ, r are two given positive constants, $f \in C([t_{k-1}, t_k) \times \mathbb{D} \times \mathbb{R}^n, \mathbb{R}^n)$, $\mathbb{D}$ is an open set in $\mathbb{PC}_r \doteq PC([-r, 0], \mathbb{R}^n)$, $\dot x$ denotes the right-hand derivative of x. For each $k \in \mathbb{Z}_+$, $I_k(t, x) \in C([t_0, \infty) \times \mathbb{R}^n, \mathbb{R}^n)$ and $I(t, 0) = 0$. For each $t \geq t_0$, $x_t \in \mathbb{D}$ is defined by $x_t(s) = x(t+s)$, $-r \leq s \leq 0$. For $\psi \in \mathbb{PC}_r^1 \doteq PC^1([-r, 0], \mathbb{R}^n)$, the norm of ψ is defined by

$$||\psi||_r^0 = \sup_{-r\leq\theta\leq0} \|\psi(\theta)\|, \quad ||\psi||_r^1 = \max\left\{ \sup_{-r\leq\theta\leq0} \|\psi(\theta)\|, \sup_{-r\leq\theta\leq0} \|\dot\psi(\theta)\| \right\}.$$

For given $\sigma \geq t_0$ and $\phi \in \mathbb{PC}_r^1$, the initial value problem of (1) is

$$\begin{cases} \dot x(t) = f(t, x_t, \dot x(t-\tau(t))), & t \in [\sigma, +\infty) \cap [t_{k-1}, t_k), \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(t_k, x(t_k^-)), & k \in \mathbb{Z}_+, \\ x_t = \phi, & \sigma - r \leq t \leq \sigma. \end{cases} \tag{2}$$

In this paper, we assume that the solution for the initial value problem (2) does exist and is unique which will be written in the form $x(t, \sigma, \phi)$, see [37–39] for detailed information. Moreover, we assume that $f(t, 0, 0) = 0$, $I_k(t, 0) = 0, t \geq \sigma, k \in \mathbb{Z}_+$ so that system (2) has a solution $x \equiv 0$, which is called the zero solution. Furthermore, in this paper we always suppose that function f satisfies the following Lipschitzian condition:

$$\|f(t, \varphi_1, \nu_1) - f(t, \varphi_2, \nu_2)\| \leq L_1(||\varphi_1 - \varphi_2||_r^0) + L_2(\|\nu_1 - \nu_2\|),$$
$$\varphi_i \in \mathbb{D}, \nu_i \in \mathbb{R}^n, \ i = 1, 2,$$

where $L_i(s), i = 1, 2$, are two monotone nondecreasing continuous functions satisfying $L_i > 0$ for $s > 0$.

We introduce some definitions (see [36,40]) as follows:

Definition 1. The function $V : \mathbb{R}_+ \times \mathbb{D} \to \mathbb{R}_+$ belongs to class ν_0 if

(i) V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{D}$ and $\lim_{(t,\varphi)\to(t_k^-,\psi)} V(t, \varphi) = V(t_k^-, \psi)$ exists;

(ii) $V(t, x)$ is locally Lipschitzian in x and $V(t, 0) \equiv 0, t \in \mathbb{R}_+$.

Definition 2. Let $V \in \nu_0$, for any $(t, \psi) \in [t_{k-1}, t_k) \times \mathbb{D}$, the upper right-hand Dini derivative of V along the solution of (2) is defined by

$$D^+V(t, \psi(0)) = \limsup_{h\to0^+} \frac{1}{h}\{V(t+h, x(t+h, \psi)) - V(t, \psi(0))\}.$$

Definition 3. A continuous function $g : \mathbb{R}_+ \to \mathbb{R}_+$ is called a *wedge function* if $g(0) = 0$, $g(s)$ is strictly increasing in $s \in [0, +\infty)$ and $g(s) \to +\infty$ as $s \to +\infty$.

Definition 4. Assume $x(t) = x(t, \sigma, \phi)$ be the solution of (2) through (σ, ϕ). Then the zero solution of (2) is said to be

(H_1) *stable,* if for any $\sigma \geq t_0$ and $\varepsilon > 0$, there exists some $\delta = \delta(\varepsilon, \sigma) > 0$ such that $||\phi||_r^1 < \delta$ implies that $\|x(t, \sigma, \phi)\| < \varepsilon$, $t \geq \sigma$;

(H_2) *uniformly stable,* if the δ in (H_1) is independent of σ;

(H_3) *uniformly asymptotically stable,* if (H_2) holds and for any $\sigma \geq t_0$, there exists some $\eta > 0$ such that $||\phi||_r^1 < \eta$ implies that $\lim_{t\to\infty} x(t, \sigma, \phi) = 0$.

3. Main results

In this section, we present some Lyapunov–Razumikhin conditions for testing the local stability and asymptotical stability of impulsive neutral system (2).

Theorem 1. *Assume that there exist functions $V(t, x) \in \nu_0$, $\mathscr{B}(t) \in \mathbb{B}$ and constants $\beta_k \in \mathbb{R}_+, k \in \mathbb{Z}_+$ such that*

(i) $u_1(\|x\|) \leq V(t, x) \leq u_2(\|x\|), t \in \mathbb{R}_+, x \in \mathbb{R}^n$, *where $u_i : \mathbb{R}_+ \to \mathbb{R}_+$ are wedge functions;*

(ii) $V(t_k, \psi(0) + I_k(t_k, \psi)) \leq (1 + \beta_k)V(t_k^-, \psi(0))$ *for $\psi \in \mathbb{PC}_r^1$, where $\sum \beta_k < +\infty, k \in \mathbb{Z}_+$;*

(iii) *For any $\sigma \geq t_0$, $\psi \in \mathbb{PC}_r^1$ and any function $N(t) \in C(\mathbb{R}_+, \mathbb{R}_+)$,*

$$D^+V(t, \psi(0)) \leq F(t, V(t, \psi(0)), N(t)),$$
$$t \in [\sigma, +\infty) \cap [t_{k-1}, t_k), \ k \in \mathbb{Z}_+,$$

whenever

$$V(t+\theta, \psi(\theta)) \leq N(t), \quad \|\dot\psi(\theta)\| \leq \mathscr{B}\big(u_1^{-1}(N(t))\big), \quad -r \leq \theta \leq 0,$$

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