



A generalized public goods game with coupling of individual ability and project benefit



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ARTICLE INFO

Article history:

Received 23 February 2017

Revised 3 May 2017

Accepted 16 May 2017

Available online 23 May 2017

Keywords:

Public goods game

Individual ability

Project benefit

Group-size preference

ABSTRACT

Facing a heavy task, any single person can only make a limited contribution and team cooperation is needed. As one enjoys the benefit of the public goods, the potential benefits of the project are not always maximized and may be partly wasted. By incorporating individual ability and project benefit into the original public goods game, we study the coupling effect of the four parameters, the upper limit of individual contribution, the upper limit of individual benefit, the needed project cost and the upper limit of project benefit on the evolution of cooperation. Coevolving with the individual-level group size preferences, an increase in the upper limit of individual benefit promotes cooperation while an increase in the upper limit of individual contribution inhibits cooperation. The coupling of the upper limit of individual contribution and the needed project cost determines the critical point of the upper limit of project benefit, where the equilibrium frequency of cooperators reaches its highest level. Above the critical point, an increase in the upper limit of project benefit inhibits cooperation. The evolution of cooperation is closely related to the preferred group-size distribution. A functional relation between the frequency of cooperators and the dominant group size is found.

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1. Introduction

The occurrence and maintenance of cooperation in a competitive setting have drawn much attention of biologists, economists, sociologists, mathematicians and statistical physicists [1–7], which is similar to the diffusion systems and the complex systems in the physical world [8–14]. In order to find the internal mechanism of the Nash equilibrium and the flourish of cooperation among completely rational individuals, evolutionary game theory and quite a few classical game models, such as the prisoner's dilemma (PD) and the snowdrift game (SG), have been employed to model the evolution of altruistic behavior [15–23]. The PD is a standard metaphor to explain the evolution of cooperation through pairwise interactions. For group interactions, the public goods game (PGG) represents a straightforward generalization of the PD. The

SG is also a cooperative game model describing pairwise interactions. The only difference between the PD and the SG is the payoff matrix. In the PD, a cooperator gets nothing. In the SG, a cooperator has a net gain after deducting the cost of cooperation. The N-person snowdrift game (NSG) represents a straightforward generalization of the SG.

In the original PGG [24,25], each individual has to decide whether to make a contribution to a common pool or not. As all the individuals have made their decisions, the total investment in the common pool is multiplied and distributed equally among all the members in the interacting group. Those who contribute to the common pool are cooperators and those who make no contribution are defectors. A rational analysis will result in such a bad scenario where nearly all the individuals contribute nothing to the common pool in order to obtain a higher personal gain and the tragedy of the commons occurs. In the PGG, each cooperator's contribution is the same and predefined. An increase in the number of cooperators in the interacting group does not change each cooperator's contribution but leads to a rise of the total cost and the

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total benefit. Different from the payoff functions in the PGG, in the NSG [26,27], having finished a task, each member of the interacting group gets the same and predefined benefit while the predefined cost is evenly shared by cooperators. Therefore, in the NSG, the total benefit increases while the total cost does not change with the rise of the group members.

However, in real society, the evolutionary mechanism in the PGG and that in the NSG may coexist [28]. The upper limit of individual ability and the upper limit of project benefit may result in such a scenario: facing a heavy task, an individual can do nothing because of his limited ability. Only when there are quite a few cooperators in the interacting group can the task be finished and the project benefit be obtained. For example, the task of moving a big and heavy rock or hunting a big creature can not be finished until there are enough cooperators in the interacting group. The impact of the critical mass or the start-up cost on the evolution of cooperation has been discussed by Szolnoki and Perc [29,30]. In addition to that, the profit from finishing the heavy task depends on the potential benefits of the project and the characteristics of the individuals who have the right to enjoy such benefits. The existence of the upper limit of individual benefit may result in such a scenario where the project benefits are partly wasted. For example, as a bridge has been built, its bearing capacity may not be fully exploited until there are quite a few individuals to-and-fro. As a group of wolves have captured a big deer, they usually enjoy a good meal and give up the leftovers. The profit from hunting a big creature can not be maximized until there are quite a few wolves in the group. As the project benefit has been maximized, an increase in the team members will lead to a decrease in the individual benefit. Although the start-up cost and the coupling of the PD game and the SG game have been discussed in refs. [29–32], the coupling effect of individual ability and project benefit on the evolution of cooperation and the coupling model of PGG and NSG are left unconsidered.

The coupling effect of different kinds of games on the evolution of cooperation is usually studied depending upon the threshold game models [30,33–36]. Szolnoki and Perc have incorporated the start-up costs into the public goods game [30]. They have found that the existence of a threshold acting as an initial contribution to the common pool can promote the levels of cooperation effectively. Perc has incorporated the success-driven mechanism into the public goods game [33]. He has found that the reproductive success of individuals promotes cooperation effectively irrespective of the interaction structure. Szolnoki and Chen have introduced a level of payoff acting as a threshold for an individual to organize the public goods game [34]. They have found that such a mechanism can keep cooperation at a somewhat high level. Chen and Perc have incorporated maximal endowments into the public goods game [35]. They have found that an excessive abundance of common resources is detrimental to cooperation. Zhang et al. have incorporated an insurance covering the potential loss into the threshold public goods game [36]. They have found that an increase in the compensation from the insurance leads to more contributions. The role of other threshold parameters in the evolution of cooperation has been discussed in refs. [37–48].

The evolution of individual strategies may not be sufficient for the occurrence and maintenance of cooperation among selfish individuals. The coevolutionary mechanism, such as the coevolution of the individual strategies and the interaction structures, may be seen as an effective way to promote cooperation. Perc and Szolnoki have reviewed the coevolutionary rules affecting the evolution of cooperation [49]. The rules of mutual interaction, population growth, reproduction, mobility, reputation and aging have a powerful effect on the evolution of cooperation.

To mimic the limitedness of individual ability and the potential benefits of the project, in the present we introduce four param-

eters, the upper limit of individual contribution, the upper limit of individual benefit, the needed project cost and the upper limit of project benefit into the original PGG. In addition to that, the evolution of the individual-level group-size preferences is also considered. Accompanied by the coevolution of the preferred group-sizes and the strategies of cooperation and defection, the coupling effect of individual ability and project benefit on the evolution of cooperation is extensively studied. We have three main findings.

(1) A higher level of individual contribution leads to a lower level of cooperation while a higher level of the upper limit of individual benefit leads to a higher level of cooperation. In the process of carrying out a heavy task with a given cost, if an individual's maximum contribution is limited, there should be more cooperators in finishing the task. The existence of the start-up cost promotes cooperation. As the upper limit of individual benefit increases, an individual is more possible to obtain a higher benefit, which leads to a higher level of cooperation.

(2) A higher level of the upper limit of project benefit leads to a lower level of cooperation. There exists a critical point of the upper limit of project benefit, below which the equilibrium frequency of cooperators changes little with the rise of the upper limit of project benefit and above which an increase in the upper limit of project benefit leads to a decrease in cooperation. The critical point is determined by the coupling of the upper limit of individual contribution and the needed project cost.

(3) A higher level of cooperation is in accordance with a smaller value of the dominant group size. The frequency of cooperators evolves with the individual-level group size preferences. The occurrence of a higher level of cooperation is accompanied by the occurrence of a smaller dominant group size. A functional relation between the equilibrium frequency of cooperators and the dominant group size is found.

The generalized public goods game (GPGG) is presented in Section 2. Simulation results and discussions are given in Section 4. In Section 5 we give a theoretical analysis on the relationship between the equilibrium frequency of cooperators and the dominant group size. In Section 5 we summarize our conclusions.

2. The model

The generalized public goods game (GPGG) is defined as follows. Assuming there is a project with the needed project cost C_{pro} and the upper limit of project benefit B_{pro}^{max} . If there are enough cooperators in the interacting group, $n_C \geq n_C^{min}$, the project can be finished and the project benefit B_{pro} depends on the upper limit of project benefit B_{pro}^{max} , the upper limit of individual benefit B_I^{max} and the number of individuals n in the interacting group. On condition that $nB_I^{max} \leq B_{pro}^{max}$, $B_{pro} = nB_I^{max}$. On condition that $nB_I^{max} > B_{pro}^{max}$, $B_{pro} = B_{pro}^{max}$. If there are not enough cooperators in the interacting group, $n_C < n_C^{min}$, the project can not be finished and the project benefit is $B_{pro} = 0$.

The threshold of the number of cooperators n_C^{min} is determined by the needed project cost C_{pro} and the upper limit of individual contribution C_I^{max} , which is satisfied with the equation $n_C^{min} = \lceil \frac{C_{pro}}{C_I^{max}} \rceil + 1$ for $\frac{C_{pro}}{C_I^{max}} > \lceil \frac{C_{pro}}{C_I^{max}} \rceil$ and $n_C^{min} = \lceil \frac{C_{pro}}{C_I^{max}} \rceil$ for $\frac{C_{pro}}{C_I^{max}} = \lceil \frac{C_{pro}}{C_I^{max}} \rceil$. For $n_C \geq n_C^{min}$, a cooperator's contribution to the project is $C_C = \frac{C_{pro}}{n_C}$ and a defector's contribution to the project is $C_D = 0$. For $n_C < n_C^{min}$, neither cooperators nor defectors make a contribution to the project, $C_C = C_D = 0$.

The value of n_C^{min} is closely related to the values of C_{pro} and C_I^{max} . The relationship between n_C^{min} and $\frac{C_{pro}}{C_I^{max}}$ is a step function. For example, within the range of $C_{pro} \leq C_I^{max}$, $n_C^{min} = 1$, within the range of $1 < \frac{C_{pro}}{C_I^{max}} \leq 2$, $n_C^{min} = 2$. Such an assumption is in accor-

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