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Multiformity of periodic-impact motions of a harmonically forced soft-impacting system and experimental verification based on an electronic circuit

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A B S T R A C T

In this paper, we consider a two-degree-of-freedom harmonically forced soft-impacting system. Basic types, characteristics and multiformity of periodic-impact motions of the system are achieved through multi-parameter simulation analyses which provide the partition of the parameter space into qualitatively different regions. The influence of the clearance, constraint stiffness, external force and forcing frequency on dynamics of the system is investigated in definite parameter spaces. The results show that the quantity of impact motions with the forcing period fully depends on the value of the constraint stiffness and such period-one multi-impact motions predominantly occur in low frequency and small clearance domain. The experiment is conducted on an electronic circuit designed according to the dynamical model of the harmonically forced system with a clearance. The outputs of the designed circuit are well consistent with the numerical results of the harmonically forced soft-impacting system, which validates the experimental approach.

1. Introduction

Vibrating systems with clearances are frequently encountered in engineering fields. Repeated impacts inevitably occur in such systems whenever their components contact the motion limiting constraints or collide with each other, but the phenomena are undesirable as the impacts bring about failures, shorter service life and increased noise levels. Being an important class of piecewise smooth dynamical systems, the studies on dynamics of vibrating systems with clearances are relevant to many applications such as chatter-impacting problem associated with piping systems and the rattle problems caused by demeshing-impact phenomena of multistage gear transmission systems or lateral wheel–rail interaction of railway vehicles. Due to the richness of nonlinear dynamical behaviors of mechanical systems with clearances, some people have devoted their researches to this subject over the years. A number of analytical and numerical studies on vibro-impact dynamics have shown that such piecewise smooth systems can exhibit all the standard dynamical behaviors which can be found in smooth nonlinear systems [1–11] and can also undergo unconventional or unique dynamical behaviors such as grazing bifurcation [12–26], sliding bifurcation [27, 28], border-collision [29] and chattering [30–32], etc. The great interest in experimental research of dynamics of piecewise smooth mechanical systems has been drawn in the past several years, which can be reflected by a still increasing amount of research literatures devoted to this aspect. Experimental methods for testing dynamical signals of practical systems with clearances or stops have been developed and several experimental devices modeling the vibro-impact systems have been designed, as partly reported in Refs. [33–41].

Actually, an experimental approach, based on the electronic circuits designed according to dynamical models of mechanical systems with clearances or rigid stops, is not only convenient and feasible, but it is also a low-cost way of verifying dynamic behaviors of such systems. The experimental approach can be well carried out and dynamical behaviors of the systems can be summarized by analyzing the intrinsic qualities of output signals generated by the circuits themselves. However, we note that most research works on implementation of electric circuits describing nonlinear dynamics have focused on Chua's circuit family [42–48], Lorenz systems [49–52] and Duffing oscillators [53, 54] in the past several years. Few people have given special consideration for the equivalent electronic circuit realization of dynamics of non-smooth mechanical systems. Zimmerman, Celaschi and Neto [55] introduced an elec-

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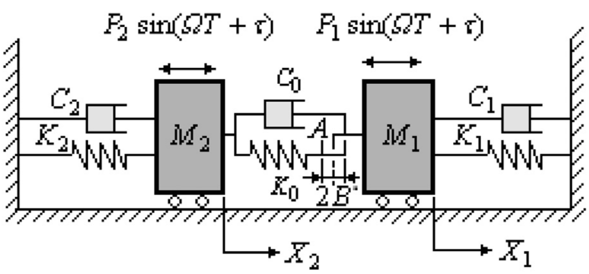

Fig. 1. Mechanical model of the two-degree-of-freedom harmonically forced system with a relative clearance.

tronic analog of the mechanical bouncing ball by use of an elementary circuit composed of three operational amplifiers with feedback current from a rectifier. Clark, Martin and Moore et al. [56] studied the fractal dimension of strange attractor of an electronic circuit modeling of a ball bouncing on an oscillating table, and doped out a solution to the problem of identifying periodic and chaotic behaviors by calculating the correlation dimension of the system from time series data taken from the circuit. Lee [57] presented a diode rectifier circuit designed according to dynamic equations of a single-degree-of-freedom mechanical oscillator impacting a rigid stop. Srinivasan, et al. [58] investigated the effect of non-smooth periodic forces like square wave, triangle wave and sawtooth wave on a driven Duffing oscillator, which was mimicked by a suitable electronic analog circuit using operational amplifiers, Miller integrators and multipliers. Ho and Nguyen [59] studied a variety of nonlinear dynamic responses for an electro-vibro-impact system, with indication of chaotic behavior.

The objectives of the present paper focus on multiformity of periodic-impact motions of a harmonically forced soft-impacting system and experimental verification based on an electronic circuit. The remainder of this paper is organized as follows. Section 2 introduces the mechanical model of the system and presents two Poincaré sections specially defined for the type and feature identification of various periodic motions. In Section 3, the influence of parameters (the clearance, constraint stiffness, external force and forcing frequency) on dynamics of the system is discussed in definite parameter spaces. In Section 4, a circuitry realization of dynamical behaviors of the system is put forward and the experimental apparatus is described. In Section 5, various periodic-impact motions of the mechanical model, obtained by numerical analysis, are experimentally verified. Last section summarizes and concludes this paper.

2. Mechanical model of a harmonically forced system with a relative clearance

Let us consider the mechanical model reported in Fig. 1, it is a harmonically forced system with the masses M_1 and M_2, the viscous damping coefficients $\bar{C}_1$ and $\bar{C}_2$, the spring stiffness coefficients K_1 and K_2. The displacements of mass blocks M_1 and M_2 are represented by X_1 and X_2, respectively. The mass block M_i is attached to the supporting base by the spring-damper elements K_i and $\bar{C}_i (i=1,\ 2)$, and subjected the harmonic force $P_i\sin(\Omega T+\tau)$. In the external forces, P_i denotes the amplitude of harmonic force acted on the mass M_i, Ω is the forcing frequency and τ is the phase angle. A clearance is put between two mass blocks, which can be modeled by a linear damper $\bar{C}_0$, symmetric constraint set with stiffness K_0 and gap value $2B$ The symmetric constraint set means that two constraints of the clearance are symmetrically situated in the distances B and $-B$ from the equilibrium position of the mass block M_1. The stiffness K_0 and gap value $2B$ determine that the symmetric constraint set plays a role of soft stops [2, 3, 20, 25]. The relative displacement of the system is restricted due to the existence of the clearance. As its relative displacement equals the gap value B or $-B$ (i.e. $|X_1-X_2|=B$), the mass M_1 begins to hit the constraint. Consequently, non-smooth nonlinearity of motion trajectory of the system inevitably appears. Correspondingly, dynamics of the system can be analyzed by a piecewise linear differential equation set switched along with the mass block M_1 touching or escaping the constraints, which can be explicitly expressed by

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \frac{\mathrm{d}^2}{\mathrm{d}T^2}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} \bar{C}_1+\bar{C}_0 & -\bar{C}_0 \\ -\bar{C}_0 & \bar{C}_0+\bar{C}_2 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}T}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{Bmatrix} F_1(X_1,X_2) \\ F_2(X_1,X_2) \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}\sin(\Omega T+\tau) \tag{1}$$

in which the constraint forces exerted by the constraint spring can be written by a piecewise linear function

$$F_1(X_1,X_2) = \begin{cases} K_0(X_1-X_2-B), & X_1-X_2>B, \\ 0, & |X_1-X_2|\le B, \\ K_0(X_1-X_2+B)\ , & X_1-X_2<-B\ . \end{cases}$$
$$F_2(X_1,X_2) = -F_1(X_1,X_2) \tag{2}$$

As for analyzing dynamics of the system in relatively large parameter spaces and making a comparison with the experimental results, it is advantageous to introduce dimensionless parameters and variables to Eqs. (1) and (2). The dimensionless variables and time are given by

$$x_i = \frac{X_iK_1}{P_1+P_2},\quad t = T\sqrt{\frac{K_1}{M_1}},\quad i=1,2. \tag{3}$$

Correspondingly, Eq. (1) can be rewritten as

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{\mu_m}{1-\mu_m} \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \frac{2\zeta}{1-\mu_{c_0}} & \frac{-2\zeta\mu_{c_0}}{1-\mu_{c_0}} \\ \frac{-2\zeta\mu_{c_0}}{1-\mu_{c_0}} & 2\zeta\left(\frac{\mu_{c_0}}{1-\mu_{c_0}}+\frac{\mu_{c_2}}{1-\mu_{c_2}}\right) \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \frac{\mu_{k2}}{1-\mu_{k2}} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} \bar{f}_1(x_1,x_2) \\ \bar{f}_2(x_1,x_2) \end{Bmatrix} = \begin{Bmatrix} 1-f_{20} \\ f_{20} \end{Bmatrix}\sin(\omega t+\tau) \tag{4}$$

where the top mark "·" denotes differentiation with respect to the dimensionless time t and the constraint forces exerted by the constraint spring are expressed as

$$\bar{f}_1(x_1,x_2) = \frac{\mu_{k_0}}{1-\mu_{k_0}}(x_1-x_2) - 0.5\frac{\mu_{k_0}}{1-\mu_{k_0}}(|(x_1-x_2)+\delta| - |(x_1-x_2)-\delta|),$$
$$\bar{f}_2(x_1,x_2) = -\frac{\mu_{k_0}}{1-\mu_{k_0}}(x_1-x_2) + 0.5\frac{\mu_{k_0}}{1-\mu_{k_0}}(|(x_1-x_2)+\delta| - |(x_1-x_2)-\delta|). \tag{5}$$

Dimensionless parameters in Eqs. (4) and (5) are given by

$$\mu_m = \frac{M_2}{M_1+M_2},\quad \mu_{k_j} = \frac{K_j}{K_1+K_j},\quad \mu_{c_j} = \frac{\bar{C}_j}{\bar{C}_1+\bar{C}_j},$$
$$\omega = \Omega\sqrt{\frac{M_1}{K_1}},\quad \zeta = \frac{\bar{C}_1}{2\sqrt{K_1M_1}},\quad \delta = \frac{BK_1}{P_1+P_2},$$
$$f_{20} = \frac{P_2}{P_1+P_2},\quad j=0,\ 2. \tag{6}$$

As the stiffness K_0 of the constraint set between two mass blocks varies from zero to infinity, the impact caused by the mass