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Conditions for topologically semi-conjugacy of the induced systems to the subshift of finite type☆,☆☆

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ABSTRACT

Let X be a compact metric space and f: $X \to X$ be a continuous map. In [14], it was shown that if a dynamical system (X, f) has strictly coupled-expanding property, then the Hyperspace dynamical system $(\mathbf{K}(X), \bar{f})$, induced by (X, f), has a subsystem which is topologically semi-conjugated to a full shift (Σ_k, σ).

In this paper, we show that under some conditions more weaker than those of [14], $(\mathbf{K}(X), \bar{f})$ has a subsystem which not only is topologically semi-conjugated to a subshift of finite type (Σ_A, σ_A), but also is bigger than the subsystem builded in [14]. Furthermore, we expand above results to the fuzzy dynamical system, extended by (X, f).

1. Introduction

A topological conjugacy (or a semi-conjugacy) between two dynamical systems implies equivalency between many of dynamical properties of the two dynamical systems, so, the investigation on topological conjugacy(or semi-conjugacy) is one of important methods in the field of dynamical system. Recently, a number of papers have been dedicated to studying of symbolic dynamical systems, and many researchers investigated topological conjugacy or semi-conjugacy between symbolic dynamical systems and other dynamical systems, because symbolic dynamical systems has good intuitiveness and a diversity of dynamical properties [2–8,10,11,13,18,19].

It is well known that a crisp system is extended in natural manner to a Hyperspace dynamical system and a fuzzy dynamical system. In this context, the main problem is to investigate the connection between dynamical properties of the crisp system and the Hyperspace system induced by it or the fuzzy system extended by it [7,9,14–16].

In [18], Zhang obtained the following result.

Theorem 1.1. *Let (X, f) be a compact dynamical system. Then (X, f) has a topological dynamical subsystem topologically conjugate to the symbolic dynamical system (Σ_k, σ) if and only if there exist pairwise disjoint closed subsets $A_1, A_2, \cdots, A_k$ in X satisfying the following two conditions*

(1) $\bigcup_{j=1}^{k} A_j \subset f(A_i), \quad i = 1, 2, \cdots, k$
(2) *For all* $(i_0, i_1, \cdots) \in \Sigma_k$, $\mathrm{card} \bigcap_{s=0}^{\infty} f^{-s}(A_{i_s}) \le 1$.

Moreover, when the above condition (2) is not satisfied, (X, f) has a topological dynamical subsystem topologically semi-conjugate to the symbolic dynamical system (Σ_k, σ).

Also, in [14] the authors had got a motive from the above theorem and proved the following theorem.

Theorem 1.2. *Let (X, f) be a compact dynamical system. If there exist pairwise disjoint closed subsets $A_1, A_2, \cdots, A_k$ in X satisfying the condition (1) of Theorem 1.1, then $\Lambda = \{\bigcap_{s=0}^{\infty} f^{-s}(A_{i_s}) : (i_0, i_1, \cdots) \in \Sigma_k\}$ is an invariant subset of the Hyperspace map 2^f, and $(\Lambda, 2^f|_\Lambda)$ is topologically semi-conjugate to the symbolic dynamical system (Σ_k, σ).*

In fact, the condition (1) of Theorem 1.1 is equivalent to strictly coupled-expansion introduced in [12]. In other words, in [14], the authors have shown that a sufficient condition for a crisp compact dynamical system, under which the Hyperspace dynamical system induced by the crisp system has a subsystem that is topologically semi-conjugated to the symbolic dynamical system (Σ_k, σ), is strictly coupled-expansion.

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In [11], the authors extended the notion of (strictly) coupled-expansion to the notion of (strictly) coupled-expansion for a transition matrix A(shortly, A-coupled-expansion), which is the same as the concept of coupled-expansion if each entry of the matrix A equals to 1. This concept of A-coupled-expansion has attracted some researchers. In particular, they considered A-coupled-expansion mainly in direction of investigating on the relationship between this concept and chaos, so recently, this concept is recognized as one of the main criteria for chaos [5,6,17]. In the other direction, some papers have devoted to investigating on topological equivalency between dynamical systems which is strictly (A-)coupled-expanding and the other dynamical systems. In [4], it was shown some necessary and sufficient conditions for dynamical systems to be topologically conjugated or semi-conjugated to the subshift of finite type (Σ_A, σ_A), in the context of Hausdorff space, moreover, the strictly A-coupled-expansion of a dynamical system is a sufficient condition for it to be semi-conjugated to the subshift of finite type (Σ_A, σ_A). As shown in Theorem 3.1.6[14], in [14], the authors proved that a dynamical system that is strictly coupled-expanding, which is a special case of strictly A-coupled-expansion, has a subsystem which is topologically semi-conjugated to the symbolic system (Σ_k, σ).

From these observations, the following reasonable problems appear: a generalization of the sufficient condition of Theorem 3.1.6, and a expansion of the subsystem $(\Lambda, 2^f|_\Lambda)$.

In [7], Kupka systematically considered about fuzzifications of discrete dynamical systems and some topologies on the fuzzy space. Also, he proved that topological conjugacy (resp., semi-conjugacy) between crisp dynamical systems is extended to topological conjugacy(resp., semi-conjugacy) between the fuzzy dynamical systems extended by them.

The main goal of this paper is to consider with the above problems, that is, to generalize the sufficient condition and result of Theorem 1.1 in [14] (See Theorem 3.1.6). Also, we want to extend our research to fuzzy dynamical systems.

The rest of this paper is organized as follows. In Section 2 we show basic definitions, notations and previous results. In Section 3.1 we investigate the above problems. We obtain weaker sufficient conditions than those of Wang and Wei [14], then the subsystem of the Hyperspace system that is builded by us contains the subsystem of Wang and Wei [14]. In Section 3.2 we generalize our research to the fuzzy dynamical system.

2. Preliminaries

Let $\mathbf{N}$ be the set of all positive integers and $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. Let (X, d) be a metric space and f: $X \to X$ be a continuous map. We call a pair (X, f) as *discrete dynamical system* and moreover if X is compact, then we call it as *compact discrete dynamical system*(shortly, *compact system*). A nonempty subset $Y \subset X$ is called (*strictly*) *invariant* for f(shortly, *f-invariant*) if $f(Y)(=) \subset Y$. If an f-invariant set Y is closed, then we call that $(Y, f|_Y)$ is a *subsystem* of (X, f). For $x \in X$, the set $\{f^n(x)$: $n \in \mathbf{N}_0\}$ is called the *orbit* of x for f, denoted by $\mathrm{Orb}_f(x)$.

If for any two nonempty open subsets $U, V \subset X$ there exists an $n \in \mathbf{N}$ such that $f^n(U) \cap V \neq \emptyset$, then we say that the dynamical system (X, f)(shortly, f) is *topologically transitive*(shortly, *transitive*) [1].

Let (X, f) and (Y, g) be dynamical systems. If there exists a surjective continuous map ϕ: $X \to Y$ such that $g \circ \phi = \phi \circ f$, then we say that (X, f)(shortly, f) is (*topologically*) *semi-conjugated* to (Y, g)(shortly, g) by ϕ. If the above map ϕ is a homeomorphism, then we say that (X, f)(shortly, f) is (*topologically*) *conjugated* to (Y, g)(shortly, g) by ϕ(or, f and g is (*topologically*) *conjugated* by ϕ).

Let $m \geq 2$ and $\mathcal{A} = \{1, \cdots, m\}$. The set $\Sigma_m = \{\alpha = (a_0, a_1, \cdots) : a_i \in \mathcal{A}, i \in \mathbf{N}_0\}$ is a compact metric space with the metric

$$\rho(\alpha, \beta) = \begin{cases} 0 & \text{if } \alpha = \beta, \\ 2^{-(k+1)} & \text{if } \alpha \neq \beta \text{ and } k = \min\{i \mid a_i \neq b_i\}, \end{cases}$$

where $\alpha = (a_0, a_1, \cdots), \beta = (b_0, b_1, \cdots) \in \Sigma_m$. The *full shift map*(shortly, *shift map*) σ: $\Sigma_m \to \Sigma_m$, defined by $\sigma(\alpha) = (a_1, a_2, \cdots)$, where $\alpha = (a_0, a_1, a_2, \cdots) \in \Sigma_m$, is a self-continuous map from Σ_m to Σ_m. The dynamical system (Σ_m, σ) is called *symbolic dynamical system* by full shift map(shortly, *full shift*). Let $A = (a_{ij})$ be an $m \times m$ matrix. The definitions of transition and irreducible matrices follow [17]. For an $m \times m$ transition matrix $A = (a_{ij})$, the set $\Sigma_A = \{(b_0, b_1, \cdots) \in \Sigma_m \mid a_{b_i b_{i+1}} = 1, i \in \mathbf{N}_0\}$ is a compact σ-invariant set. The map $\sigma_A = \sigma|_{\Sigma_A}$ is called the *subshift map of finite type* for the matrix A and the subsystem (Σ_A, σ_A) is called the *subshift of finite type* for the matrix A.

Definition 2.1. [11]. Let (X, d) be a metric space and f: $D \subset X \to X$. Suppose that $A = (a_{ij})$ is an $m \times m$ transition matrix for some $m \geq 2$. If there exist m nonempty subsets Λ_i $(1 \leq i \leq m)$ of D with pairwise disjoint interiors such that

$$f(\Lambda_i) \supset \bigcup_{\substack{j \\ a_{ij}=1}} \Lambda_j$$

for all $1 \leq i \leq m$, then the map f is said to be *A-coupled-expanding* in Λ_i $(1 \leq i \leq m)$. Moreover, the map f is said to be *strictly A-coupled-expanding* in Λ_i $(1 \leq i \leq m)$ if $d(\Lambda_i, \Lambda_j) > 0$ for all $1 \leq i \neq j \leq m$, where $d(\Lambda_i, \Lambda_j)$ denotes the distance between two sets Λ_i and Λ_j. If all entries of the matrix A are equal to 1, then (strictly) A-coupled-expanding map f is shortly called (strictly) *coupled-expanding* map.

2.1. Induced Hyperspace dynamical system

First let's introduce some topologies of Hyperspace [2,15].

Let X be a topological space and $\mathcal{F}, \mathcal{G}, \mathcal{K}$ denote respectively the sets of all closed, open and compact subsets of X ($\emptyset \in \mathcal{F}$, $\emptyset \in \mathcal{G}$, $\emptyset \in \mathcal{K}$). The hit-or-miss topology τ_f on $\mathcal{F}$ is generated by the subbase

$$\mathcal{F}^K, K \in \mathcal{K}; \quad \mathcal{F}_G, G \in \mathcal{G},$$

where $\mathcal{F}^K = \{F \in \mathcal{F} : F \cap K = \emptyset\}$ and $\mathcal{F}_G = \{F \in \mathcal{F} : F \cap G \neq \emptyset\}$. A topological base of τ_f is

$$\mathcal{F}^K_{G_1, G_2, \cdots, G_n}, K \in \mathcal{K}, G_i \in \mathcal{G} (1 \leq i \leq n), n \geq 0,$$

where $\mathcal{F}^K_{G_1, G_2, \cdots, G_n} = \mathcal{F}^K \cap (\bigcap_{i=1}^n \mathcal{F}_{G_i})$. Note that $\mathcal{F}^\emptyset = \mathcal{F}$ and $\mathcal{F}^K_{G_1, G_2, \cdots, G_n}$ means $\mathcal{F}^K$ when $n = 0$.

Let (X, d) be a metric space. For any point $x \in X$ and a nonempty bounded closed subset $K \subset X$, a given $\varepsilon > 0$, we define $d(x, K) = \inf\{d(x, y) : y \in K\}$ and $N(K, \varepsilon) = \{x \in X : d(x, K) < \varepsilon\}$. The Hausdorff metric DH_X on the family of all nonempty bounded closed subsets of X is defined by

$$DH_X(K, L) = \inf\{\varepsilon > 0 : K \subset N(L, \varepsilon) \text{ and } L \subset N(K, \varepsilon)\},$$

where K, L are nonempty bounded closed subsets of X.

Denote $\mathbf{K}(X) = \{K \subset X : K \text{ is nonempty compact.}\}$. If (X, d) is a compact metric space, then the hit-or-miss topology τ_f and the topology induced by the Hausdorff metric DH_X are both compact and consistent on $\mathbf{K}(X)$. In this paper, when (X, d) is a compact metric space, we call the compact metric space $(\mathbf{K}(X), DH_X)$ as *Hyperspace*.

Define a self-continuous map $\bar{f}$ on $(\mathbf{K}(X), DH_X)$ by $\bar{f}(K) = f(K)$ for $K \in \mathbf{K}(X)$. We call the $\bar{f}$ *induced map* from f and f is called *crisp map* for $\bar{f}$. The compact dynamical system $(\mathbf{K}(X), \bar{f})$ is called *induced dynamical system*(or *induced Hyperspace dynamical system*) from (X, f).