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# Generalized dislocated lag function projective synchronization of fractional order chaotic systems with fully uncertain parameters

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In this paper, we propose a new method to improve the safety of secure communication. This method uses the generalized dislocated lag projective synchronization and function projective synchronization to form a new generalized dislocated lag function projective synchronization. Moreover, this paper takes the examples of fractional order Chen system and Lü system with uncertain parameters as illustration. As the parameters of the two systems are uncertain, the nonlinear controller and parameter update algorithms are designed based on the fractional stability theory and adaptive control method. Moreover, this synchronization form and method of control are applied to secure communication via chaotic masking modulation. Many information signals can be recovered and validated. Finally, simulations are used to show the validity and feasibility of the proposed scheme.



## 1. Introduction

Since its discovery in 1990 [1], synchronization of chaotic systems has drawn much attention due to its wide application in various fields of physics and engineering. The synchronization problem in chaotic systems has been intensively and extensively studied in recent decades [2–4].

Due to the chaotic signal is analogy randomicity, the initial value sensitivity and continuous broadband spectrum etc, which is considered to have a huge market potential for the secure communications. Chaotic signals are typically broadband, noiselike, and difficult to predict, they can be used in various contexts for masking information bearing waveforms. The secure communication method based on chaotic is a kind of dynamic encryption method, which is different from traditional encryption methods. This method has high computational efficiency and can be used for real-time signal processing. Information encrypted with a chaotic encryption method has higher confidentiality, which is difficult to decode. A variety of chaotic communication methods were built based on the synchronization method, such as chaos masking, chaotic parameter modulation, chaotic spread spectrum and chaotic shift keying. The basic idea of all these chaotic secure communication is the same. There are two steps: transmission information encryption and transmission information decryption. Transmission information encryption is putting the information source to the chaotic signal produced by a chaotic system and generating mixed noise signal. Transmission information decryption is using a corresponding chaotic system to separate chaotic signal of mixed noise signal after mixed signal sent to the receiver. The information source is restored. Chaos synchronization is the key to achieve the encryption and decryption process.

Chaotic synchronization has been discussed theoretically and experimentally. In order to improve the security of secure communication, many synchronization methods have been proposed such as complete synchronization [1], phase synchronization [5], anticipated synchronization [6], generalized synchronization [7], inverse generalized synchronization [8], projective synchronization [9], function projective synchronization [10]. Complete synchronization is defined as the coincidence of states of interacting systems; phase synchronization means entrainment of phases of chaotic oscillators whereas their amplitudes remain uncorrelated; lag synchronization which appears as a coincidence of time-shifted states of two systems; anticipated synchronization also appears as a coincidence of shifted-in-time states of two coupled systems; generalized synchronization which appears as two chaotic systems are said to be synchronized if a functional relationship exists between the states of the two systems; inverse generalized synchronization which is defined as the inverse of generalized synchronization; projective synchronization allows two chaotic systems to be synchronized up to a constant transformation called scaling factor; and func-

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tion projective synchronization is an improved version of projective synchronization and uses scaling factor function matrix instead of the scaling factor. Different synchronization types have the main difference in the controller. The more complex a synchronization strategy is, the more conducive it is for secure communication.

Most of the work in chaos synchronization has been concentrated on integer-order chaotic systems rather than fractional-order chaotic systems. In recent years, researchers discovered that many real-world physical systems can be accurately described by fractional-order differential equations [11,12]. For examples, Meerschaert described fractional governing equations that can be solved to obtain the probability densities of the limiting process which is useful to characterize the natural variability in price in the long term [13]. Hall and Barrick estimated a fractal dimension to categorize the complexity of the excursions of diffusing spins [14]. In addition, fractional differential equations have also been successfully applied in biology, physics, chemistry, hydrology, finance and other disciplines [15]. Fractional order differential equation means that the differential order time can be arbitrary or fraction equation. Fractional order differential equation extends describing ability of the integer order differential equation, which is a generalization of the integer order differential equation. Integer-order differential equation only depends on local characteristics of the function. And fractional order differential equation considers the whole information of the function by weighted form. The fractional order mathematical model can more accurately describe the dynamic response of the actual system [16]. Fractional order system is more universality and has larger keys space. Stability analysis and solution of the fractional order differential equations are far more complicated than the integer order differential equation. The more complex fractional-order chaotic synchronization scheme can further ensure the security of communication. The synchronization of fractional-order chaotic systems has become an active research field due to its potential applications in secure communication and control processing. Very recently, there have been some developments on the synchronization of fractional-order chaotic systems [17–20].

However, in the real chaotic system, due to various kinds of interferences, parameters may be uncertain or drift with time as there is always some delay time in the chaotic synchronization between the response system and the drive system. The study of time-delay chaotic systems or chaotic systems with fully uncertain parameters has more real significance. For examples, You et al. studied the sufficient conditions for lag synchronization of two different chaotic systems with time delay [21]. A kind of binary function projective lag synchronization of uncertain chaotic systems with stochastic perturbation is proposed by Xu [22].

In this paper, generalized dislocated lag function projective synchronization (GDLFPS) of fractional chaotic systems with fully uncertain parameters is proposed. The principle of chaotic secure communication based on GDLFPS is shown as Fig. 1. $m(t)$ is the transmission signal. $X = [x_1, x_2, \cdots, x_n]$ is the drive system and $Y = [y_1, y_2, \cdots, y_n]$ is the response system. $f(\bullet)$ is the key function. $s(t)$ is mixed noise signal. $\boldsymbol{u}$ is the undetermined controller vector. $\Delta \in R^{n \times n}$ is a scaling factor function matrix. $\delta_{ij}(t)(i = 1, 2, \cdots, n; j = 1, 2, \cdots, n)$ are the elements of $\Delta$, which is a time dependent function and $\tau$ is the delay time.

It can be seen from Fig. 1 that Chaos synchronization is the key to achieve the encryption and decryption process. At the receiving end, when the nonlinear controllers are built to realize the generalized dislocated lag function projective synchronization between the drive system and response system, the secure communication can be realized. Which means the more complex synchronization method, the harder decipher the secrecy communication. And it can also show that the synchronization applied secrecy communication has the following advantages:

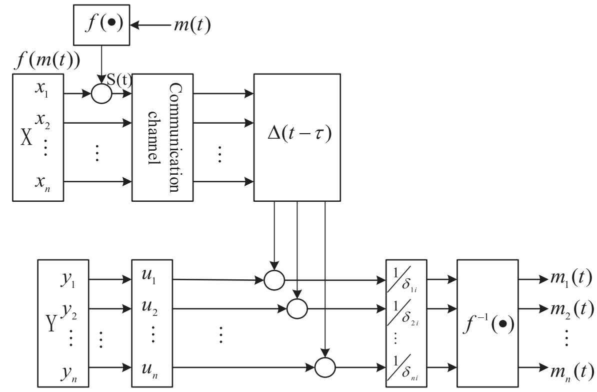


**Fig. 1.** The principle of chaotic secure communication based on GDLFPS.

1) Due to the unpredictability trajectory of integer order chaotic systems, the encryption signal is not easy to crack. Fractional order chaotic systems can through changing the fractional order to further increase the unpredictability of trajectory. So the more complex fractional-order chaotic synchronization scheme can further ensure the security of communication.
2) Different synchronization type means that the form of the scaling factor is different, and the more complex and changeable scale factor will be more conducive to the security of secret communication [23]. The generalized function projective synchronization means that scaling factor is replaced by scaling factor function matrix. Generally, the elements of scaling factor function matrix can be any function. And all elements can be replaced arbitrarily. GDLFPS can demodulate multiple signals. Through the contrastive analysis of the signal, we can judge whether the demodulation signal real and effective. If the demodulation signals eventually equal, demodulation signals are real and effective. This will further enhance the degree of confidential communication.
3) Dislocated synchronization is said to occur when all of the state variables of the response system are formed by complex non-linear combination of the variables of the drive system. This will increase the anti-crack ability of secure communications.
4) As the time delay and uncertain parameters, which are much closer to the real situation and more accurate to describe the synchronization between the transmitter and the receiver. It is more accurate and much closer to the real situation. This is one natural advantage in theory.

We use GDLFPS to achieve the synchronization between two delay time fractional chaotic systems with fully uncertain parameters and identify the unknown parameters using stability theory and parameter adaptive law. In order to show the effectiveness of the proposed scheme, the projective synchronization schemes for Chen fractional-order chaotic system and Lü fractional-order chaotic system are discussed. The results indicate the validity and feasibility of the proposed method. Moreover, this synchronization form and method of control are applied to secure communication via chaotic masking modulation. Many information signals can be recovered and validated.

The plan of the rest of the paper is as follows: In Section 2, the fractional order definition, GDLFPS and the controller designing are described and the controller is designed. In Section 3, firstly, the controller and parameter identification laws of Chen and Lü systems are established based on the theory in Section 2. Then the simulations are done for the chaotic systems. And the results