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Evidence of nonlinearity in presence of external forcing and magnetic field in a glow discharge plasma

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We implement different statistical analysis techniques to investigate the presence of nonlinearity in the fluctuations of a steady state glow discharge plasma (GDP). We introduce a novel 'delay vector variance' analysis (DVV) to allow reliable detection of nonlinearity in a plasma in presence of external forcing and magnetic field by providing some easy to interpret diagrams. The analysis has also been implemented to study numerically simulated results for the first time. An estimate of the Zscore has been carried out to detect the presence of nonlinearity in a somewhat quantitative sense. An informal test for bicoherency has been applied to detect the interaction amongst different modes obtained by performing emperical mode decomposition to strengthen the analysis on nonlinearity. The results obtained from bicoherency test resemble those obtained by performing DVV. An attempt to model the experimental observations by a second order nonlinear ordinary differential equation derived from the fluid equations of plasma has revealed convincing results.

1. Introduction

Time series analysis [1] is of extraordinary importance for the comprehension and the characterization of a physical process in order to gain relevant information about the system of interest such as in geoscience [5], medicine, biology [2] or especially complex plasma system. Natural phenomenon are often studied through measures of physical observables changing over time. Improved methods for time-series analysis may also yield sublime diagnostic information about the system under study. Things like predictability, nonlinearity [3,4] have gained ample attention from various areas of science like physiology and earth science [13]. So we are in need of verifying the existence of underlying nonlinear process to convey information dealing with the absence or presence of nonlinearity. The analysis of a time series should be able to detect the stochastic or deterministic nature of the underlying process, the presence of nonlinearity or non-stationarity [6] and finally the predicability of the future states.

Plasma effects are finding ever increasing applications in astrophysics, solid state physics, physics of gas discharge and research on plasma confinement and heating. Plasmas are intrinsically nonlinear whose effects manifest in the form of various exotic structures such as double layers [7], solitons, vortices, different types of waves, instabilities and turbulence [8]. Glow discharge plasma being rich in high energy, electrons and ions are capable of exhibiting many such nonlinear phenomenon [9–11]. GDPs and their counterparts like magnetrons [12] are widely used in industrial applications and hence characterizing them through nonlinear techniques can help in improving their performances.

It is for the first time in GDP system that we deploy some new well established nonlinear techniques like Zscore [15], bicoherency analysis [23], delay vector variance (DVV) method [37] to explore nonlinearity [16] by creating a number of surrogate data [20,25] by iterative amplitude adjusted Fourier transform method (IAAFT) [26,27] yielding time series with amplitude spectra identical to that of the original time series and approximately identical signal distributions. DVV analysis resting on the theory of time delay embedding has been introduced to study data from GDP and implemented on both experimental and numerical results.

Koepke et al. [28] have identified a periodic nonlinear interaction between pairs of self-excited, propagating, ionization waves simultaneously present in the positive column of a neon glow discharge with no external oscillatory driving force. Nurujjaman et al. [29] studied the rich dynamical behavior of an excitable glow-discharge plasma under the subthreshold and supra-threshold periodic forcing. Pardo et al. [30] have studied the effect of pacing on plasma keeping it in periodic as well as chaotic regime. Experimental results on real time phase synchronization of paced chaotic plasma discharge have been reported by Ticos et al. [31]. Current and plasma potential in a magnetized thermionic plasma

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discharge have been investigated by Klinger et al. [32]. Investigation of nonlinearity using harmonic detection method [11,18,19] has been performed in RF discharge and glow discharge plasma respectively. Although a comparative study of magnetized and unmagnetized plasma oscillations has been conducted [33] in a direct current (DC) glow discharge plasma, the qualitative and quantitative measure of nonlinearity has been performed first time in our work by varying the forcing amplitude, frequency and magnetic field. If the characteristic measure of original time series are significantly different from those for the surrogate data [34], the null hypothesis that the data can be described by linear stochastic model can be rejected. The proposed method yields a standardized characterisation of a time series that examines the local predictability over different scales. Through the study of bicoherency analysis [22] we gain information about the interaction amongst different modes obtained after performing empirical mode decomposition analysis [23,24].

In Section 2, we present a brief schematic of the experimental setup, followed by the results of the analysis of the floating potential fluctuations (FPF's), power spectral analysis in Section 3. Sections 4 and 5 represent respectively a comprehensive analysis of nonlinearity with surrogate data as well as with delay vector variance method. Section 6 features the interaction amongst different modes using bicoherency analysis. In Section 7, a numerical modelling of the experimental observations has been attempted using a forced nonlinear dynamical equation representing the temporal dynamics of ion acoustic oscillations in presence of both ionization and recombination terms. Conclusions are presented in Section 8.

2. Experimental setup

The experiments were conducted in a cylindrical hollow cathode glow discharge argon plasma with a typical density and temperature of $\sim 10^7/\mathrm{cm}^3$ and 2–4 eV respectively. It has a cylindrical cathode of length and diameter $\sim$ 17 cm and $\sim$ 10 cm, respectively, and a central anode rod of diameter $\sim$ 1.6 mm. The whole assembly was mounted inside a vacuum chamber and was pumped down to a pressure of about 0.001 mbar using a rotary pump. The chamber was subsequently filled with argon gas up to a predetermined value of neutral pressure by a needle valve. Finally a discharge was struck by a direct current discharge voltage, which could be varied in the range of 0–1000 V. As the anode was kept grounded, the applied voltage is basically negative. An unbiased Langmuir probe was used to obtain the floating potential fluctuations acquired with a sampling time of 0.2 µs, 0.5 µs respectively for 1, 1.5 kHz frequency. The tip of this Langmuir probe was placed in the center of the electrode system.

The study of nonlinearity was carried out under two different conditions (i) application of magnetic field (ii) forcing the system by using an external oscillating electric field. To carry out the observations in presence of magnetic field, an external magnetic field (axial) was applied to the plasma by passing a steady current through the coils wound over the cylindrical chamber as shown in the right panel of Fig. 1. The schematic of the measurement circuit for floating potential measurement is also depicted in Fig. 2 which shows that the probe is connected to the oscilloscope having input resistance and capacitance of the order of 1 MΩ and 13pF respectively. The magnetic field was varied from 15 to 105 Gauss for these experiments. A glow discharge plasma operating under steady state conditions is subjected to an oscillatory external voltage that constitutes forcing. For this, a signal generator was coupled with the discharge voltage (DV) through a capacitor for observing fluctuations as shown in the left panel of Fig. 1. We have applied a sinusoidal forcing of the type $F = A\sin(\omega t)$ where A is the amplitude of forcing and ω is the frequency. For two values of ω (1, 1.5 kHz), A was varied from 0.2 V (min) to 8 V (max) to study the effects of forcing on the system.

3. Floating potential fluctuation, power spectral analysis

With the plasma in the period 2 regime (4, 8 kHz dominant frequencies) for P = 0.085 mbar we have paced the system with a periodic forcing (1kHz) in Fig. 3 keeping Discharge Voltage (DV) fixed at 343 V and went on increasing the amplitude of forcing (A). Initially at low amplitude of forcing (A = 2 V) the position of the peaks in power spectrum plot shifts to 4.5 and 8.5 kHz depicted in the right panel of Figs. 3 and 4. Subsequent raising in the value of A = 5 V, the system undergoes a transition from double period to 3 period fluctuations with the introduction of a new peak at 1.5 kHz frequency. Further increase in the value of A results in generating new peaks at frequency of 5.5, 9.5 kHz. The amplitudes of the peaks generated don't linearly increase with increase in the value of forcing amplitude (A).

A three dimensional analog of the Fig. 3 is also presented in the form of contour plot in Fig. 4 with colour axis representing the power. We have carried out another set of experiment for P = 0.180 mbar by pacing the plasma in the periodic regime with the 1.5 kHz frequency. The amplitudes of the power at different values of forcing amplitude (A) are plotted in the right panel of Fig. 5. For initial increase in A upto 3 V only the frequencies of the dominant peak (peaks carrying maximum power) exist with the addition of 1.5 kHz frequency. Subsequent increase in A results in adding new frequencies (3, 3.7, 3.1 kHz) for A = 4.6, 5.4 V apart from the dominant one and the frequency by which we are pacing the plasma as seen from the amplitudes of FFT depicted in Fig. 5.

Now with the application of magnetic field the fluctuations and its corresponding power spectrum plots are portrayed in Fig. 6. Initial increase in magnetic field (B) = 15G has little effect in changing the power spectrum. As we go on increasing the magnetic field the amplitudes as well as the frequency of the spectra seem to increase as indicated by the appearance of broad band which is thought to be emerged due to the nonlinear interaction between the frequencies generated. At a very high value of magnetic field of B = 105G, we obtain the frequencies lying in the range upto 4 kHz. So we obtain maximum values of frequencies lying in kHz regime which is much less than the cut off frequency of the low pass filter combined between the sheath resistance ($R_{sh} \sim 0.2M\Omega$) and stray capacitance ($C_s \sim$ in order of tens of pF).

Dimension of a system is defined as the power of the radius of the hypersphere (l) with which the volume of the system within the hyper sphere changes. Correlation dimension (CD) [35,36] is a measurable parameter similar to the dimension of a system [14,17]; the only difference is that here we count data points of the system as we increase the volume of the hypersphere. It actually tells us what is the dimension of the nonlinear interaction whose 1D projection is obtained in the time series. To calculate the CD of the time series, we first need to define the dimension of space in which we are going to perform the calculation; or, in other words, we need to find out the embedding dimension(m) of the system. For this purpose, the correlation integral is computed as the following equation. $C(l)$ is the correlation integral in a particular embedding dimension. The correlation dimension, can be taken as the slope of the (ln (C(l); ln (l))-curve where l is a length measure which is varied. $\| X_i - X_j \|$ is the distance between the ith and jth point in that embedding space

$$C(l) = \lim_{N\to\infty} \frac{1}{N^2}\{\text{Number of pairs for which } \| X_i - X_j \| < l\} \qquad (1)$$

So the estimation of correlation dimension of the plasma floating potential signals at various magnetic fields shows that the