Chaos, Solitons and Fractals 98 (2017) 88–100

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Chaos generalized synchronization of coupled Mathieu-Van der Pol and coupled Duffing-Van der Pol systems using fractional order-derivative

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ARTICLE INFO

Article history:
Received 18 August 2016
Revised 21 February 2017
Accepted 7 March 2017

Keywords:
Chaos
Synchronization
GYC partial region stability theory
Lyapunov function
Adam–Bashforth–Moulton method

ABSTRACT

In the present paper, the synchronization by the Ge-Yao-Chen (GYC) partial region stability theory of chaotic Mathieu-Van der Pol and chaotic Duffing-Van der Pol systems with fractional order-derivative is proposed. Numerical simulations show that this synchronization technique is very effective and it turns out that the fractional order-derivative induces quick synchronization compared to integer order-derivative of these systems. In order to bring out the chaotic behavior of these systems either with fractional or with integer order-derivative, we simulate their phase portraits and the Lyapunov exponent. Moreover, we provide in this work an approximated solution to both systems to show that the solution of such a system can be represented as a simple power-series function. Furthermore, the representation of the error dynamics with respect to the time before and after the control action approves the effectiveness of the control method and proves the possibility of stabilization and controllability of chaotic systems with an appropriate. Furthermore, the synchronization of the fractional Mathieu-Van der Pol system using the fractional Duffing-Van der Pol system is simulated.

1. Introduction

The properties of non-linear systems have a great deal of interest, since they have many applications in various fields of sciences such as in biological, chemical and physical systems. The dynamics of such systems might be specified by various different means including ordinary differential equations which are the most known, partial differential equations, iterate maps and recently fractional differential equations. *The last notion involves the derivative with memory and it is the generalization of ordinary and partial differential equations*. Typical examples of systems that can be represented by these general types of equations are onset of coherent radiation in lasers and masers [1], self-excitations in electric circuits, self-organizations in chemical reactions [2], non-linear mechanics [3], etc.

The occurrence of chaotic behavior in non-linear oscillators subjected to periodic forcing is widespread and well known. Examples are the Duffing equation arising almost ubiquitously in models of mechanical oscillations [4], the Van der Pol equation describing, for example, the triode oscillation in electrical circuits [5] and the Mathieu equation which describes for example the motion of particles vibrating in an elliptic drum [6], which have been extensively studied. The parameter space of these equations are divided with great complexity into regions of different qualitative behavior, and the space of initial states is divided with similar complexity into the basin of attraction of competing attractors which may be steady, periodic or chaotic. The last notion describes erratic motions in non-linear dynamical systems. Nowadays, the chaos theory is used in several domains such as geophysics, meteorology, astronomy, economy, biology, etc. Moreover, the chaos theory is the best mechanism for signal design with potential application in telecommunication and coding systems [7]. A chaotic system is unpredictable but it is perfectly described by deterministic equations [8]. It is deterministic because, knowing the exact state of a system at some given time: the initial state can help define the state of the system at any time. Deterministic and unpredictability are two paradoxical notions but the link between them is determined by the sensitivity to initial conditions [9]. That means

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http://dx.doi.org/10.1016/j.chaos.2017.03.012
0960-0779/

two almost initial conditions can lead to very different states of the system. The impossibility to predict the evolution of deterministic system is a real characteristic of chaotic systems. Due to their unpredictability character, chaotic phenomena are very difficult to control. However, scientists have shown that a chaotic motion can be controlled under certain conditions. Even so, our main interest in this work is the study of the chaotic or hyperchaotic systems described by fractional order-derivative systems. It has been shown recently that, chaotic behavior can appear with fractional order differentiation [10]. Over the unpredictability of such a system, its instability character makes its controllability a big challenge. However, because of its several potential applications, many researchers are nowadays interested and have even succeeded to control chaotic motions either theoretically or experimentally [11]. Hence, the synchronization of chaotic systems has gained increasing attention after the pioneer work of Pecora et al. [12] in 1990. That is, many types of chaos synchronization have been proposed such as the phase synchronization developed by Pecora in [12]. The complete synchronization has also been developed by Liu et al. in [13], the adaptive synchronization by Shihua et al. [11]. Furthermore, Zheng-Ming et al. [14] developed the generalized synchronization using the GYC (Ge-Yao-Chen) partial region stability theory for the case of Mathieu-Van der Pol and Duffing-Van der Pol systems. Added to this, we have also the projective synchronization [15], hybrid synchronization [16], etc. Generally, these types of synchronization have been carried out using several synchronization schemes such as: linear and non-linear feedback synchronization [17], adaptive control [11], time delay feedback approach [18], etc. In their work, Hongmin et al. [19] have shown that for the order of derivative $\alpha = 0.9$, the system behaves chaotically. In the same idea, Ghaderi et al. [20] studied the control and synchronization of chaotic Coullet system using fractional order-derivative. They used active control for synchronization and the simulations show the effectiveness of the method. Very recently, Kumar et al. [21] introduced a Mathieu-Van der Pol system with fractional order-derivative. They came out with some remarkable conclusions: the synchronization of Mathieu-Van der Pol chaotic system of fractional order-derivative by linear feedback can be achieved and then, the stability of the system is possible under certain conditions. However, they did not take into consideration the more general case where we have different order-derivatives in the system which implies $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$.

In this paper, the coupled Duffing-Van der Pol and the coupled Mathieu-Van der Pol chaotic systems with fractional order-derivative are studied taking into consideration this generality. The synchronization is approached by the GYC partial region stability theory. This implies by theorem that if V is a positive definite function on the partial region with opposite sign to that of its derivative and the function V itself permits an infinitesimal upper limit, then the undisturbed motion is asymptotically stable on the partial region [14]. Compared to other techniques of synchronization which include the backstepping method, the adaptive design method, the linear and non-linear feedback method, sampled-data feedback synchronization, time-delay feedback, etc. [22–25], this techniques is very appropriate due to the fact that it introduces less simulation error in the system, also from the fact that using this theory we are able to construct the Lyapunov function as a simple linear-function from where the control parameters are easily designed. This synchronization approach is more general since it is applicable for the autonomous and non-autonomous systems, for perturbed and unperturbed systems and finally for linear and non-linear systems. The upper drawbacks can be overcome by using this method. This drawback can only be calculated in the case of finite evolution time in computer simulation. However, infinite evolution time is needed by definition of Lyapunov exponent [26]. It has been applied to several chaotic systems and the simulation results demonstrate the effectiveness and feasibility of the method. These systems include the fly-ball governor with and without system structure perturbation [26], Lorenz system [26],etc. These are the motivations for choosing to apply this synchronization technique to the fractional Mathieu-Van der Pol and fractional Duffing-Van der Pol systems in this work.

The present paper is structured as follows: In Section 2, the coupled Mathieu-Van der Pol and the coupled Duffing-Van der Pol systems with fractional order-derivative are introduced. In Section 3, we provide an approximated solution to the Mathieu-Van der Pol and Duffing-Van der Pol fractional systems using the Variational Iterative Method. In Section 4, we introduce the stability analysis, the synchronization scheme and the fractional Lyapunov exponent is presented. Section 5 presents the numerical results. Finally, the conclusion is provided in Section 6.

2. Coupled Mathieu-Van der Pol and the coupled Duffing-Van der Pol systems with fractional order-derivative

The coupled Mathieu-Van der Pol system [21] with fractional order-derivative is given as follows:

$$\begin{cases} D_*^{\alpha_1} x_1(t) = x_2 \\ D_*^{\alpha_2} x_2(t) = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 \\ D_*^{\alpha_3} x_3(t) = x_4 \\ D_*^{\alpha_4} x_4(t) = -ex_3 + f\left(1 - x_3^2\right)x_4 + gx_1, \end{cases} \tag{2.1}$$

with initial values $x_1(0)$, $x_2(0)$, $x_3(0)$, $x_4(0)$ and a, b, c, d, e, f, g the parameters of the system. For all the simulations in this paper, we considered (a, b, c, d, e, f, g)=(10, 3, 0.4, 70, 1, 5,0.1) and the initial conditions (x_{10}, x_{20}, x_{30}, x_{40})=(0.1, -0.5, 0.1, -0.5). Let us now consider $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$ all real numbers taken in (0,1). Fig. 1 gives the phase portrait of the system plotted in **Python** with different values of α so that, $\alpha_1 = 0.98,\ \alpha_2 = 0.99,\ \alpha_3 = 0.999,\ \alpha_4 = 0.99$. Analogically, the Duffing-Van der Pol equation with fractional order derivative is given by:

$$\begin{cases} D_*^{\alpha_1} z_1 = z_2 \\ D_*^{\alpha_2} z_2 = -z_1 - z_1^3 - hz_2 + iz_3 \\ D_*^{\alpha_3} z_3 = z_4 \\ D_*^{\alpha_4} z_4 = -jz_3 + k\left(1 - z_3^2\right)z_4 + lz_1, \end{cases} \tag{2.2}$$

where h, i, j, k, l are also the parameters of the system with the values (h, i, j, k, l)=(0.0006, 0.67, 1,5, 0.05) and the initial conditions (z_{10}, z_{20}, z_{30}, z_{40})=(2, 2.4, 5, 6).

Let us now consider the fractional order-derivative where $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$ are all real numbers taken in (0,1), As in the previous case the system also exhibits chaotic behavior which is shown in Fig. 2 depicting the phase portrait of the system for different order of derivative, $\alpha_1 = 0.98,\ \alpha_2 = 0.99,\ \alpha_3 = 0.999,\ \alpha_4 = 0.99$. $D_*^{\alpha_i},\ \ 0 < \alpha_i < 1$ stands for the Caputo derivative defined so that, for $n - 1 < \alpha < n,\ \ n \in \mathbb{N},\ \ \alpha \in \mathbb{R}$ and a function $f(t)$ such that $D_*^{\alpha} f(t)$ exists, one has

$$D_*^{\alpha} f(t) = \frac{1}{\Gamma(n - \alpha)} \int_a^t (t - s)^{-1-\alpha+n} f^{(n)}(s) ds, \tag{2.3}$$

where $f^{(n)}(s)$ denotes the ordinary derivative of order n of the function $f(s)$.

From Figs. 1 and 2, we observed that, as well as the Mathieu-Van der Pol and the Duffing-Van der Pol using integer-order derivative, the fractional order derivative of these systems also depicts a chaotic behavior. This is the case for the values of α_i in the range of [0.9-1[. These curves are the so called attractor from their character to keep a particle or an object in the same region no matter where the particle starts its motion.