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Letter to the editor

Bifurcation and chaos of electromechanical coupling main drive system with strongly nonlinear characteristic in mill

1. Introduction

One major reason for the damage of rolling mill equipment is torsional vibration [1]. The vibration problems will become worse as the steel strip get thinner and the rolling speed get higher. In addition to the effects of torsional vibration on the stability of control systems, the damage to rolling mill itself is also very serious. The torsional vibration can easily cause fatigue and damage of transmission parts, which is one of the main reasons why machine parts can't be used longer. Violent vibration can also induce sudden destructive fracture and lead to great economic losses [2,3]. In the production process of rolling mill at home and abroad, the accidents that torsional vibration leads to the damage of driving parts and has implications for the normal production of the rolling plant often occur [4,5].

There are many research works on torsional vibration in rolling mills. Li et al. [6] investigated the dynamics of a single degree of freedom system for the rolling mill with clearance, and analyzed the effects of boundary conditions on the vibrating system. Brusa et al. [7] investigated the effectiveness of commercial multi-body dynamics codes in predicting the dynamic behavior of cold rolling mill. Sun et al. [8] studied the stability of some 1660 mm tandem rolling mill with different cases. It was found that the rolling speed and the thickness of strip have strong influences on the stability of parametric resonances. Tamaoki et al. [9] analyzed the transfer function of variable-speed rolling mill motor with shaft torsional vibration systems. Kim et al. [10] proposed a mathematical model of cold rolling mill to predict chatter vibration. Hou et al. [11] established a vertical-horizontal coupling nonlinear vibration dynamic model of rolling mill rolls based on the interactions between the dynamic rolling force and mill structure.

The mechanical vibrations of drive systems and electric current vibrations in the motor windings were regarded as mutually uncoupled in the traditional research [6–11]. While it is particularly significant to take the electromechanical coupling effects into consideration when exact results are required for investigation of extremely responsible drive systems or for analysis of their sufficiently precise and stable motions [12–16]. This problem has been already studied for many years, but in majority of cases sufficiently accurate electromechanical models are not usually used. Tang et al. [17] investigated the unsteady condition of rolling mill vibration caused by flexural-vibration of the strip. The characteristics of the electrical current in a temp driving motor's main loop were studied and tested. Zhong et al. [18] established a global coupling dynamic model of a temper mill system. Coupling properties were depicted and its effects were analyzed. Xiang et al. [19,20] established the mathematical model of an asymmetric rotor-bearing system, considering nonlinear oil-film force and rub-impact force. Nonlinear coupled dynamics of the system was investigated. The results indicate that crack depth, stator stiffness and eccentricity have influences on the vibration and instability of the system. The dynamic processes of electromechanical coupled vibration were simulated with the changes of current regulator parameters, damping, harmonic disturbance, gap, load disturbance, and so on in reference [21]. Yan et al. [22] obtained the characteristics and rules of vibration using self-made comprehensive telemetry system and analyzed the test signal in the time and frequency domain. The results confirmed that the nature of the rolling mill vibration is mechanical-electrical-liquid coupling vibration. Liu et al. [23,24] established the nonlinear dynamic equation of coupled electromechanical drive system of rolling mill based on Lagrange-Maxwell theory. The effect of electromagnetic factors on nonlinear dynamic performance of the electromechanical coupling system was investigated. Szolc et al. [25] proposed an analytical-computational approach for electromechanical coupling investigations, which enables the producers of various machine drive systems to make right choices of optimal asynchronous motors for driven objects.

Rolling mill drive system is a huge system, involving many disciplines such as steel rolling, machine, motor, automatic control and power supply. The system will produce electromechanical vibration and even broken strip when the machine and electricity can't match. In fact, the main drive system of rolling mill is a typical strongly nonlinear electromechanical coupling dynamic system [26]. The mill electromechanical vibration is ubiquitous, but in academic circles few scholars research the strongly nonlinear torsional vibration of rolling mill's electromechanical coupling system. In this paper, a strongly nonlinear torsional vibration model of the main drive system in rolling mill under electromagnetic excitation was established by means of Lagrange–Maxwell equation. The stability and bifurcations of the periodic solutions for the system with varying excitation amplitudes based on the incremental harmonic balance (IHB) method and Floquet theory were investigated. The route from order to chaos was identified semi-analytically. The evolution of the Floquet multipliers under varying excitation amplitudes was used to predict the bifurcations and chaos. The results presented in this paper contain both the quantitative and qualitative information.

2. The electromechanical coupling dynamic model for mill

Rolling mill is a complicated vibration system, which consists of many subsystems with nature vibration characteristics and among them main transmission system plays a crucial role. The method of lumped mass is adopted to abstract the rolling mill main drive system driven by AC synchronous motor into a two-mass model, as shown in Fig. 1.

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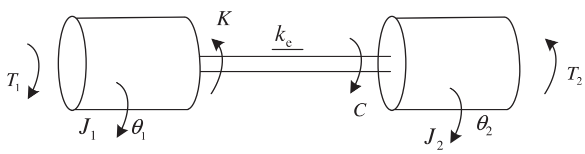

Fig. 1. Dynamic model of rolling mill's main drive system.

The total kinetic and potential energy of the system are given by

$$E = \sum_{i=1}^{2} \frac{1}{2} J_i \dot{\theta}_i^2 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2, \tag{1}$$

$$V = \frac{1}{2} K(\theta_1 - \theta_2)^2, \tag{2}$$

where $J_i (i = 1, 2)$ are the moment of inertia of the system; K is the linearity torsional rigidity of the system; $\theta_i (i = 1, 2)$ and $\dot{\theta}_i (i = 1, 2)$ are the angle of rotation and rotational speed, respectively.The system Lagrange function is

$$L = E - V = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 - \frac{1}{2} K(\theta_1 - \theta_2)^2, \tag{3}$$

The generalized torque $Q_i (i = 1, 2)$ are given by

$$Q_i = \sum_{i=1}^{2} F_i^i \frac{\partial \theta_i}{\partial q_j} (j = 1, 2), \tag{4}$$

where, $F_i^i = T_i + F_i^c$, F_i^c are generalized damping force, T_i are generalized external torque, and $q_j (j = 1, 2)$ are generalized coordinates.

The air gap magnetic field energy between the stator and rotor is

$$W = \frac{R_1 L}{2} \int_0^{2p\pi} \Lambda(\alpha, t) \cdot F^2(\alpha, t) d\alpha, \tag{5}$$

where R is the stator inner radius. L is the rotor effective length.

The salient pole synchronous motor air gap permeance can be expressed as follows:

$$\Lambda(\alpha, t) = \Lambda_0 [1 + \frac{1}{2}(x^2 + y^2) + x cos\alpha + y sin\alpha + x y sin2\alpha + \frac{1}{2}(x^2 - y^2) cos2\alpha], \tag{6}$$

where $x = \varepsilon \cos\gamma, y = \varepsilon \sin\gamma, \Lambda_0 = \frac{\mu_0}{k_\mu \delta_0}$, ε is the equivalent length of air gap eccentricity. γ is the angular displacement of rotor. μ_0 is the magnetic permeability of air. k_μ is the magnetic saturation. δ_0 is the equivalent length of air gap. According to the theory of electrical machine, the fundamental synthetic magnetomotive force (mmf) of stator and rotor can be obtained:

$$F(\alpha, t) = F_{sm} \cos(\alpha - \omega t) + F_{jm} \cos(\alpha - \omega t - \frac{\pi}{2} - \varphi + \psi - p\phi_1), \tag{7}$$

where F_{sm} and F_{jm} are the amplitudes of stator and rotor fundamental mmf, respectively. ω is the network angular frequency. α is the electrical angle. ψ is the internal power angle. φ is the power factor angle. p is the number of pole-pairs. ϕ_1 is the torsional vibration angle of rotor. The electromagnetic torque can be expressed as:

$$T_1 = \frac{\partial W}{\partial (p\phi_1)}, \tag{8}$$

Substituting Eqs. (5), (6) and (7) into Eq. (8), the electromagnetic torque expression of salient pole synchronous motor can be derived as follows:

$$T_1 = b_0 + b_1 \phi_1 + b_2 {\phi_1}^2 + b_3 {\phi_1}^3, \tag{9}$$

in which, $b_0 = -pR_1 L \Lambda_0 F_{sm} F_{jm} \pi \cos(\varphi - \psi)$, $b_1 = p^2 R_1 L \Lambda_0 F_{sm} F_{jm} \pi \sin(\varphi - \psi)$,

$$b_2 = \frac{p^3 R_1 L \Lambda_0}{2} F_{sm} F_{jm} \pi \cos(\varphi - \psi),$$

$$b_3 = -\frac{p^4 R_1 L \Lambda_0}{6} F_{sm} F_{jm} \pi \sin(\varphi - \psi).$$

The system generalized damping force ${F_1}^c$, ${F_2}^c$ are:

$${F_1}^c = -C(\dot{\theta}_1 - \dot{\theta}_2), {F_2}^c = C(\dot{\theta}_1 - \dot{\theta}_2). \tag{10}$$

Using the dissipation Lagrange equation

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} + \frac{\partial F}{\partial \dot{q}_i} = Q_i, \tag{11}$$

the following Lagrange's equations of motion are obtained:

$$\begin{cases} J_1 \ddot{\theta}_1 + K(\theta_1 - \theta_2) + C(\dot{\theta}_1 - \dot{\theta}_2) = T_1, \\ J_2 \ddot{\theta}_2 + K(\theta_2 - \theta_1) + C(\dot{\theta}_2 - \dot{\theta}_1) = -T_2, \end{cases} \tag{12}$$

where T_1 is the motor torque, and T_2 is the load torque.

Let

$$T_1 = T_{10} + \Delta T_1, T_2 = T_{20} + \Delta T_2, \tag{13}$$

where T_{10}, T_{20} are constant compositions of motor torque and load torque, respectively. ΔT_1, ΔT_2 are disturbance compositions of motor torque and load torque, respectively. Suppose θ_{10}, θ_{20} are torsional angles at two ends of shaft caused by T_{10} and T_{20}, respectively. ϕ_1, ϕ_2 are torsional vibration angles at two ends of shaft, respectively. Then

$$\begin{array}{l} \theta_1 = \theta_{10} + \phi_1, \theta_2 = \phi_{20} + \varphi_2, \\ \dot{\theta}_{10} = \dot{\theta}_{20}, \ddot{\theta}_{10} = \ddot{\theta}_{20} = 0. \end{array} \tag{14}$$

Substituting Eqs. (13) and (14) into Eq. (12), one can obtain

$$\begin{cases} J_1 \ddot{\phi}_1 + K(\Delta\theta + \phi_1 - \phi_2) + C(\dot{\phi}_1 - \dot{\phi}_2) = T_{10} + \Delta T_1, \\ J_2 \ddot{\phi}_2 + K(\phi_2 - \phi_1 - \Delta\theta) + C(\dot{\phi}_2 - \dot{\phi}_1) = -T_{20} - \Delta T_2 - T_f. \end{cases} \tag{15}$$

When the system is running at a constant torque, we can get $\phi_1 = \phi_2 = 0$. Substituting it into Eq. (15) yields

$$\begin{cases} K\Delta\theta = T_{10}, \\ K\Delta\theta = T_{20} + T_{f0}, \end{cases} \tag{16}$$

where T_{f0} is constant composition of T_f.

Substituting Eq. (16) into Eq. (15) and considering the load disturbance torque in the form of $\Delta T_2 = F\cos\omega t$, the system equivalent nonlinear dynamic equations on the strength of the load disturbance moment can be expressed as

$$\begin{cases} J_1 \ddot{\phi}_1 + K(\phi_1 - \phi_2) + C(\dot{\phi}_1 - \dot{\phi}_2) - K_e \phi_1 - b_2 {\phi_1}^2 - b_3 {\phi_1}^3 = 0, \\ J_2 \ddot{\phi}_2 + K(\phi_2 - \phi_1) + C(\dot{\phi}_2 - \dot{\phi}_1) + F\cos\omega t = 0. \end{cases} \tag{17}$$

Define the coefficient of linear term in T_1 as electromagnetic stiffness K_e, that is, $K_e = b_1$ [27]. The expression of K_e, $K_e = p^2 R_1 L \Lambda_0 F_{sm} F_{jm} \pi \sin(\varphi - \psi)$, shows that the electromagnetic stiffness will change with F_{sm} (proportional to the stator current), F_{jm} (proportional to the exciting current), and ψ.

According to the practical parameters of electromechanical coupling torsional vibration system for 2150 rolling mill in a certain factory, $K_e = b_1 = 1.92 \times 10^5$ N•m/rad, $b_2 = -5.76 \times 10^5$ N•m/rad^2, $b_3 = 3.84 \times 10^5$ N•m/rad^3 and $K = 1.13 \times 10^7$ N•m/rad can be calculated. It is not difficult to find that the nonlinear term coefficients b_2 and b_3 are not small compared with the linear torsional rigidity K, and greater than the linear electromagnetic rigidity K_e. So the system is obviously an electromechanical coupling system with