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Frontier

# A prediction method based on wavelet transform and multiple models fusion for chaotic time series

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ABSTRACT

In order to improve the prediction accuracy of chaotic time series, a prediction method based on wavelet transform and multiple models fusion is proposed. The chaotic time series is decomposed and reconstructed by wavelet transform, and approximate components and detail components are obtained. According to different characteristics of each component, least squares support vector machine (LSSVM) is used as predictive model for approximation components. At the same time, an improved free search algorithm is utilized for predictive model parameters optimization. Auto regressive integrated moving average model (ARIMA) is used as predictive model for detail components. The multiple prediction model predictive values are fusion by Gauss–Markov algorithm, the error variance of predicted results after fusion is less than the single model, the prediction accuracy is improved. The simulation results are compared through two typical chaotic time series include Lorenz time series and Mackey–Glass time series. The simulation results show that the prediction method in this paper has a better prediction.



## 1. Introduction

Chaos is a kind of irregular movement, which is widely existed in nature. It is a complex behavior generated by a certain nonlinear dynamical system. The characteristics of chaotic signal make it more and more important in the field of signal processing, communication, control, social economy, and biomedicine, etc. With the research of chaos theory and application technology, the modeling and prediction of chaotic system has become an important research topic in the field of information processing in recent years [1,2]. Chaotic time series prediction is an important research topic in chaotic systems. It is widely used in weather prediction [3], power load forecast [4] financial stocks forecast [5], traffic flow prediction [6], runoff forecast [7] and etc. In nature and human activities, a lot of time series are nonlinear and even chaotic, so how to accurately predict the chaotic time series is very important.

The prediction of chaotic time series can be regarded as an inverse problem in the study of dynamical systems. The positive problem is to study the various properties of the orbits in the phase space of a given nonlinear dynamical system. The inverse problem is how to construct a nonlinear mapping function to express the original system, which is a series of iterative sequences or a set of observation sequences in the given phase space. This mapping can be seen as a predictive model. Therefore, how to construct the prediction model is a key problem in chaotic time series prediction [8]. In recent years, many scholars have used many methods to model and predict the chaotic time series system. Such as linear prediction model (including AR [9], ARMA [10], ARIMA [11], etc.), neural network model (including RBF neural network [12], echo state [13], extreme learning machines [14], Elman neural network [15]), support vector machine [16], least square support vector machine [17], adaptive filter [18] and etc. At the same time, many scholars have used some combination prediction model to predict the chaotic time series [19–21].

But there is a major problem in the relevant research results at present. The research shows that the chaotic time series contains the noise signal, which will destroy the inherent dynamic characteristics of the system and reduce the prediction accuracy. Therefore, it is necessary to reduce the noise of chaotic time series. Wavelet transform is a good tool to solve the problem of noise reduction. Although some scholars have considered this problem, using wavelet transform, empirical mode decomposition and other methods to reduce the noise of chaotic time series. According to the characteristic of each component, the prediction is carried out by independent [22–25]. However, the results of each model prediction are a direct superposition, so that the prediction errors of each prediction model can be accumulated to the final prediction value, and can not further improve the prediction accuracy. Based on the above discussions, this paper proposes a novel chaotic time

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series prediction method based on wavelet transform and multiple models fusion. Firstly, wavelet decomposition and single reconstruction are carried out for non stationary chaotic time series. The single reconstruction ensures that the signal is consistent with the length of the original signal. The original chaotic time series is filtered into a smooth approximate component sequence through wavelet decomposition and reconstruction. The approximate component reflects the trend of the original time series, so the LSSVM with good prediction ability is used to predict the nonlinear time series. At the same time, an improved free search algorithm (IFS) is used to optimize the parameters of the LSSVM prediction model. For the filtered noise sequence, the detail component has a certain random non-stationary characteristic. Therefore, this paper uses the ARIMA model to predict the detailed components. When calculating the final predictive value, this paper does not directly add the predictive value of each component. The multiple model fusion prediction is carried out by Gauss–Markov estimation algorithm. The predictive error variance after fusion is smaller the single model predictive error variance, which makes the prediction accuracy is further improved. Finally, through the simulation of two typical chaotic time series — Lorenz time series and Mackey–Glass time series show that the proposed prediction method has higher accuracy.

## 2. Wavelet transform

Wavelet transform uses orthogonal basis to decompose the signal [26]. Discrete wavelet transform is composed of a series of parameters.

$$c_j(k) = \langle X, \varphi_{jk}(t)\rangle, d_j(k) = \langle X, \psi_{jk}(t)\rangle, j, k \in Z \quad (1)$$

Where, $\langle *, *\rangle$ in the above equation is the inner product, $c_j(k)$ as the approximation component, $d_j(k)$ as the detail component, the scaling function $\varphi_{jk}(t)$ is obtained from the mother wavelet $\varphi(t)$ after shifting and stretching.

$$\varphi_{jk}(t) = 2^{-j/2}\varphi(2^{-j}t - k) \quad (2)$$

$\varphi_j(t)$ is a low-pass filter, the low frequency component of the input signal can be separated. Wavelet transform can decompose a signal into a set of detail components with large scale and approximation components with different small scale.

In this paper, the fast discrete orthogonal wavelet transform Mallat algorithm [20] is used for decompose and reconstruct for chaotic time series.

$a_j$ will be considered as a chaotic time series for decomposition, according to decomposition algorithm:

$$a_{j+1} = Ha_j, \quad d_{j+1} = Gd_j, \qquad j = 0, 1, 2 \cdots N \quad (3)$$

$H$ is a low pass filter and $G$ is a high pass filter. Through the Mallat algorithm, the original chaotic time series sequence is decomposed into approximation components with low-frequency and detail components with high-frequency. Approximate components can reflect the changing trend and characteristics of time series. The detail components reflect the dynamic factors of disturbance. The chaotic time series after decomposed can be reconstructed by Mallat algorithm, the algorithm is as follows:

$$a_{j-1} = a_jH^* + d_jG^*, j = 0, 1, 2, \cdots N \quad (4)$$

Where, $H^*$ and $G^*$ are dual operator of $H$ and $G$. Mallat reconstruction algorithm uses two interpolations, which is the input chaotic time series between every two adjacent sequence zero. The algorithm can keep the decomposition and reconstruction of sequence length consistent.

Daubechines wavelet has very good characteristics for non-stationary time series, but the dbN wavelet with different $N$ values has different processing effect. The greater $N$ is, the computation time is longer. Db3 wavelet is used according to experiments and references. The decomposition level is mainly related to SNR (signal to noise ratio), when the SNR is low, the input signal is mainly noise, and then decomposition level should choose bigger. It is conducive to the separation of signal and noise. When the SNR is high, the input signal is mainly signal, then decomposition level shouldn't choose bigger. Decomposition level is too large will lead reconstruction distortion is more serious, the error will be large. In this paper, the three decomposition and reconstruction is used to ensure real-time and prediction precision [27–28].

## 3. The multiple model fusion prediction

### 3.1. ARIMA prediction model

The basic idea of ARIMA model for non-stationary time series uses several difference operation to make it become a stationary series, the number of differential is $d$. The ARMA model with $p$, $q$ as parameters is used for modeling the stationary time series. The original time series is obtained after inverse transform. ARIMA model prediction equation with $p$, $d$, $q$ as parameters can be expressed as:

$$y_t = \theta_0 + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \cdots \theta_q\varepsilon_{t-q} \quad (5)$$

Where, $y_t$ is the sample value of the time series, $\varphi_t$ and $\theta_t$ are the model parameters, $\varepsilon_t$ is independent normally distributed white noise.

After the time series is smoothed, the auto correlation function (ACF) and the partial correlation function (PCF) of the original time series is first calculated. For a time series $y_t$, there is auto covariance:

$$\gamma_k = \frac{1}{N}\sum_{j=1}^{N-k} y_k y_{t+k} \quad (6)$$

Auto correlation function:

$$\rho = \frac{\gamma_k}{\gamma_0} \quad (7)$$

Partial correlation function:

$$\left\{\begin{array}{l} \alpha_{11} = \rho_1 \\ \alpha_{k+1,k+1} = (\rho_{k+1} - \sum \rho_{k+1-j}\alpha_{kj}) \\ \qquad \times (1 - \sum_{j=1}^{k} \rho_j \alpha_{kj})^{-1} \\ \alpha_{k+1,j} = \alpha_{kj} - \alpha_{k+1,k+1} \times \alpha_{k,k-j+1} \end{array}\right\} \quad (8)$$

The model order can be determined through cutoff property of $\rho_k$, $\alpha_k$. Parameter identification of time series can be obtained by least squares estimation. Through the parameters estimation of $\varphi_1$, $\varphi_2$, $\cdots$, $\varphi_p$, $\theta_1$, $\theta_2$, $\cdots$, $\theta_q$, it makes the following equation minimum.

$$\sum_{t=1}^{N} \alpha_t^2 = \sum_{t=1}^{N} (\theta_q^{-1}(Z)\varphi_p(Z)\nabla^d y_t)^2 \quad (9)$$

Where, $\alpha_t^2$ is the sum of squared residuals, $Z$ is backward shift operator, $y_t$ is original time series, $N$ is the length of time series, $\nabla^d$ is $d$ order difference operator, $\varphi_1$, $\varphi_2$, $\cdots$, $\varphi_p$,$\theta_1$, $\theta_2$, $\cdots$, $\theta_q$ are parameters to be estimated. The (9) shows that these parameters $\varphi_1$, $\varphi_2$, $\cdots$, $\varphi_p$, $\theta_1$, $\theta_2$, $\cdots$, $\theta_q$ can be estimated when the minimum sum of squared residuals are 1obtained. Then $y_t$ can be predicted by (5) through historical data $y_{t-1}, \cdots, y_{t-p}$.

The combination of different parameters $p$, $d$ and $q$, the parameters of the optimal model is obtained by AIC (Akaike information criterion) [29]. AIC criterion characterized as stingy principle of concrete, defined as follows:

$$\text{AIC} = -2\ln L + 2n \quad (10)$$