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Review

Current reversal in a symmetric periodic potential

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ABSTRACT

In previous research work, investigators have focused mainly on noise-induced current reversal (CR) in spatially ratchet systems, while less effort has been devoted to the CR in a spatially symmetric periodic potential. Transport of an underdamped Brownian particle in a symmetric periodic potential driven by correlated noises (correlation between additive noise and multiplicative noise) and periodic signals is investigated. By means of numerical calculations, we obtain average velocity (current) of the system. The results indicate that motion direction of the particle changes with increment of the additive or multiplicative noise intensity in case of the correlated noises, i.e., CR. Our further investigations show that the additive signal and the correlation between the noises are two necessary ingredients for the CR in the spatially symmetric system. And the CR is brought about by the symmetry breaking induced by the noises, not by temporal symmetry breaking.

1. Introduction

In the past decades, current reversal (CR), i.e., the current changes its direction in certain parameter regions of model, has been studied intensively. And it has been used to model many different situations such as particle separation, intracellular transport and Josephson junction transport, etc [1–5]. Opening out essential mechanisms and conditions of CR has represented a formidable challenge for scientists for a long time. So far, current reversal has been mostly observed in ratchet systems [6–15]. A nonlinear mixing of signals which lead to temporal symmetry breaking can make current be inverse in spatially symmetric potentials [16–19]. If temporal symmetry isn't broken, the proper deformation of a spatially periodic potential can also induce CR [20]. In addition, CR also depends on the interplay between potential asymmetry, noise, driving frequency, and inhomogeneous friction [21]. Researches on CR in symmetric periodic systems have referred to focus on correlation time of thermal noise [22] or dissipation-induced symmetry breaking [23,24]. Actually, apart from the factors, correlation between additive noise and multiplicative noise can also induce CR in symmetric periodic potential, which will be discussed in this paper. For positive (negative) correlation, direction of the current changes from positive (negative) to negative (positive) with the increment of additive and multiplicative noise intensity, i.e., CR. In addition, the current as function of the noise intensities exhibits a peak (valley) at smaller noise intensities and a valley (peak) at large noise intensities.

It's well known that noise always has internal and external origins. In some physical systems [25,26], internal noise (additive noise) and external noise (multiplicative noise) can come from thermal perturbation and external environmental perturbation (e.g., electromagnetic fields , external vibrations, etc.), respectively. For a system driven by noises, we can simultaneously consider the additive noise and the multiplicative noise. Moreover, in certain situations the two noises may have a common origin and thus may be correlated with each other [27–29]. Under the action of correlated noises, dynamic properties of a system often exhibit more abundant and interesting [30–32]. One of the most intriguing results is that as the correlated strength between noises increases, the stationary probability distribution of a bistable system changes from a bimodal to a unimodal structure [33]. Furthermore, the efficiency of the ratchet system can be maximized in the presence of correlated noises [34]. In addition, the correlation between multiplicative and additive noises also induces directional motion of a particle in symmetric periodic potentials [35]. The reason for the above results is that the correlated noises coerce the potential tilted in one direction, and break the symmetry of the potential, satisfying necessary condition of current emergence. A periodic system, however, often possesses several asymmetries. If correlated noises and time-asymmetric drivings are both presented in a spatially symmetric periodic system, what statistical properties the system's current will present, as far as we know, has not been fully understood yet.

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In this paper, we will investigate abnormal transport of an inertial Brownian particle in a system with a spatially symmetric potential and signals under the action of correlated noises, and discuss conditions and physical mechanisms of the CR in the system. The paper is constructed as follows: In Section 2, model and theoretical analysis are provided. In Section 3, results and discussions are presented. By means of numerical calculations, effects of the external drivings and the correlated noises on the current will be discussed. In Section 4, conclusions are made.

2. Model and theoretical analysis

Here, we study transport of an inertial Brownian particle subjected to typical correlated noises [29,35], which is modeled by the following equation:

$$\ddot{x} + \gamma \dot{x} = -V'(x) + F(t) + \eta(t) + \cos(2\pi x)\xi(t), \tag{1}$$

where x is the state variable, the dot above x represents the derivative of x with respect to time t, and γ is the friction coefficient. Previous work has focused mainly on transport of a particle in ratchet periodic potentials [7–14]. In what follows, we will consider the case of symmetric periodic potential with the form:

$$V(x) = \sin(2\pi x). \tag{2}$$

Symmetry breaking is one of the necessary conditions that a net current appears in a periodic system. For the spatially symmetric potential considered here, symmetry breaking of the system can come from temporal asymmetry. Generally, mixing of two signals with different frequencies results in the temporal symmetry breaking [22,23]. So we assume that time-asymmetry driving has following form:

$$F(t) = A[\cos(\omega t) + \varepsilon \cos(2\omega t + \phi)], \tag{3}$$

where A and ε are two constants, ϕ is the initial phase. As $\varepsilon = 0$, the system is temporally symmetric; otherwise asymmetric.

$\eta(t)$ and $\xi(t)$ in Eq. (1) are additive and multiplicative noises, respectively. Sometimes, the noises are not independent, but correlated to each other. For example, in the Josephson junctions [26], the multiplicative noise and the additive noise come from external environmental perturbations (e.g., the perturbation of electromagnetic fields or external vibration) and thermal perturbation, respectively. The environmental perturbation can also lead to change of the thermal vibration of molecules in the Josephson junction. Such fluctuation effects of the molecular vibrations will affect the additive noise, which leads to the correlation between them [33,36,37]. Thus $\eta(t)$ and $\xi(t)$ are assumed to be correlated, and satisfy the following statistical properties:

$$\begin{aligned} \langle \xi(t) \rangle &= \langle \eta(t) \rangle = 0 \\ \langle \xi(t)\xi(t') \rangle &= 2D\delta(t-t'), \\ \langle \eta(t)\eta(t') \rangle &= 2\alpha\delta(t-t'), \\ \langle \xi(t)\eta(t') \rangle &= \langle \xi(t')\eta(t) \rangle = 2\lambda\sqrt{D\alpha}\,\delta(t-t'), \end{aligned} \tag{4}$$

where α and D are the noise intensities, and λ is the noise correlation strength.

In the underdamped case, current of the particle is difficult to find analytically. But we can use Eqs. (1) and (4) to calculate its mean velocity. In process of numerical calculations, the correlated noises can't be treated with directly, so it is very necessary to develop a transformation, i.e., a decoupling scheme [38], or a stochastic equivalent method [33]. Here, the correlated noises can decoupled by the following transformations:

$$\xi(t) = \sqrt{D} w_1(t), \tag{5}$$

$$\eta(t) = \lambda\sqrt{\alpha} w_1(t) + \sqrt{\alpha(1-\lambda^2)} w_2(t), \tag{6}$$

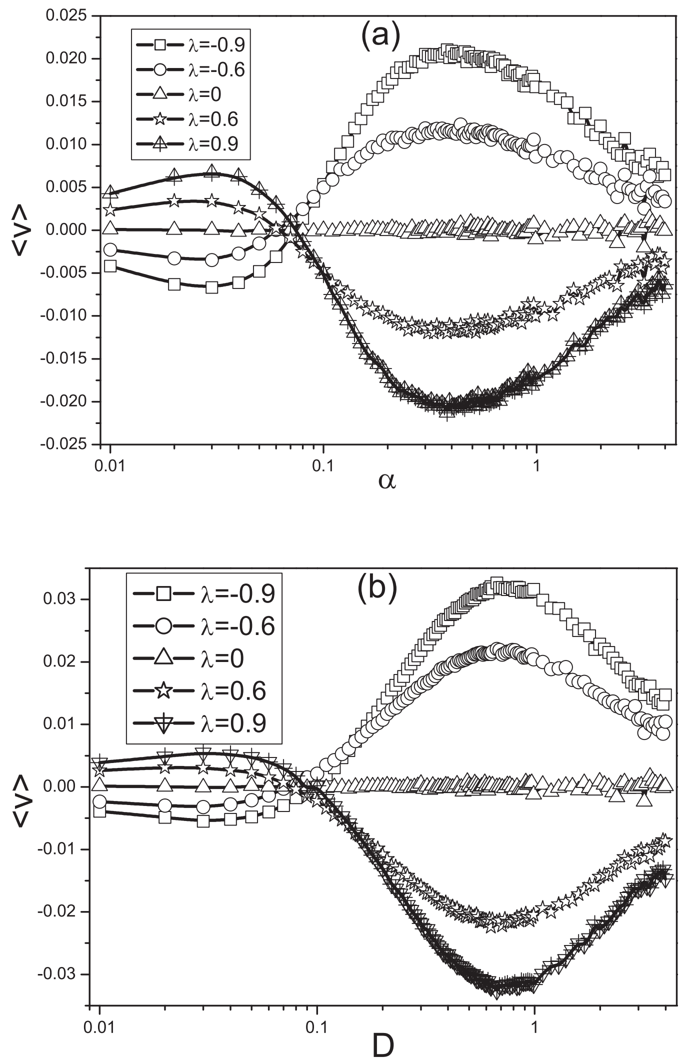

Fig. 1. The dependence of the mean velocity $\langle v \rangle$ on the additive noise intensity α for (a) and the multiplicative noise intensity D for (b) at different values of the correlation strength λ: $\lambda = -0.9, -0.6, 0, 0.6, 0.9$, with time step 0.001. The other parameters are $A = 4.2$, $\omega = 4.9$, $\phi = 0 \varepsilon = 0$, $\gamma = 0.9$, $D = 0.05$ for (a) and $\alpha = 0.05$ for (b).

where $w_1(t)$ and $w_2(t)$ are two independent Gaussian white noises with zero mean and unit variance.

The time step is chosen so small (e.g., 0.001) that the errors brought by Euler algorithm can be accepted. Then we can easily calculate the mean velocity (or current) of the particle according to its definition

$$\langle v \rangle = \lim_{t \longrightarrow \infty} [x(t) - x(0)]/t, \tag{7}$$

where $\langle\rangle$ denotes ensemble average. In the hypothesis of ergodicity, the time average is equal to the ensemble average. In the process of computing $\langle v \rangle$, 500 different trajectories were run and each trajectory evolved for $t = 2 \times 10^4$.

3. Results and discussions

By means of Eqs. (1) and (5)–(7), we can calculate numerically the mean velocity of the particle, and the results were plotted in Figs. 1–6. In the spatial symmetric periodic system, there exist two types of symmetry breaking: one is the temporal breaking caused by $F(t)$, the other is the breaking due to the correlation between $\eta(t)$ and $\xi(t)$ in Eq. (1). In what follows, we mainly discuss effects