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Modeling opinion dynamics: Theoretical analysis and continuous approximation

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ABSTRACT

Frequently we revise our first opinions after talking over with other individuals because we get convinced. Argumentation is a verbal and social process aimed at convincing. It includes conversation and persuasion and the agreement is reached because the new arguments are incorporated. Given the wide range of opinion formation mathematical approaches, there are however no models of opinion dynamics with nonlocal pair interactions analytically solvable. In this paper we present a novel analytical framework developed to solve the master equations with non-local kernels. For this we used a simple model of opinion formation where individuals tend to get more similar after each interactions, no matter their opinion differences, giving rise to nonlinear differential master equation with non-local terms. Simulation results show an excellent agreement with results obtained by the theoretical estimation.

1. Introduction

In group discussions individuals exchange arguments over a specific subject of conversation, and then selectively either incorporate what they have discovered or at least learn to understand one another better. That is to say, individuals may want to change their own opinions about an issue in order to get closer to or farther from others in the group. These interactions give rise to the formation of different kinds of opinions in a society. At the end of the discussion the group will be characterized either by a so called opinion consensus or coexistence of opinions (fragmentation). The processes of opinion formation and opinion change have always been under the close supervision for modeling. Until now various approaches exist and they all differ in their focus and complexity (see for instance, [1–5]). We recently published a new threshold model of opinion formation [6], in which the opinion change emerges as a consequence of a persuasion interacting dynamics between convinced agents, or between convinced and undecided agents; and a repulsion effect occurs whenever the agents belong to opposite groups. The model has been studied through simulations, and we showed that the system presents a wide spectrum of solutions, as a function of the fraction of undecided individuals and the adjustment in the individual's persuasion after interaction. We achieved to derive the masters equations that govern the process of opinion formation dynamics. These equations, a nonlinear coupled system of first order differential equations of hyperbolic type with nonlocal terms, are driven by two competitive terms representing two ubiquitous mechanisms in opinion formation: agreement and negative influence. They are of special interest for their nontrivial properties but they are very hard to being solved numerical or analytically. There are few models of this type, even for a single equation. For instance, in [3] where only agents with similar opinion can interact, the nonlocal terms involve a small neighbourhood of a given opinion and they simplify them by performing Taylor expansions. With this approach they recover local equations of Fokker-Planck type, but this is only possible in the frame of bounded confidence models and the long range interactions are lost. In [7], the authors deal with a model of opinion formation where nonlocal terms are not simplified, but they involve a coupling between each individual opinion and the mean of the opinions. As far as we know, there are no models of opinion dynamics with nonlocal pair interactions analytically solvable. A logical step then is to face this problem focusing in one of the main mechanism involved in most of the opinion models [8]: persuasion interacting dynamics and the compromise hypothesis. In order to proceed and work out the analytical framework we reduced the

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original model [6] to a single population, where whenever two individuals interact, their opinions get changed by a fixed discrete quantity. We obtain a continuous approximation of the master equation that rule the evolution of the system, and in this case, it is possible to solve it explicitly using a method developed by Li and Toscani [9]. This method permits to find the exact solutions of the continuous approximation of the master equation, which then are compared with numerical simulations. Let us mention that the same idea was applied by Aletti et al. [7] to a different model of opinion dynamics, where a first order equation was derived and the mean opinion of the population appears as a coefficient of the drift. Here, we get a kind of nonlocal Porous Media type equation, which can be thought as a first order hyperbolic equation with a nonlinear, nonlocal flux. This partial differential equation develops a shock at the median of the distribution, and the median value moves toward the mean. We show that the distribution of individual's opinions converges to a Dirac's delta function concentrated at the mean opinion of the initial distribution. Let us mention that introducing the bounded confidence hypothesis and restricting the interaction to sufficiently close agents, the equation converges to a Porous Media equation backward in time similar to the ones appearing in [3]. However, in this case we obtain an ill-posed problem, lacking the continuity with respect to small perturbations of the initial data or the solutions, and this explains why the system is difficult to analyze from both the numerical and theoretical point of view. There exist few theoretical results and numerical methods for these problems, which are currently being under active research. What we observe is that we can obtain an analytical solution that can be useful to solve more complex problems where this dynamics is present, such as for instance [5,6]. The paper is organized as follows. First we present the model and derive the master equations. Then, we derive the solutions, compare them with the numerical model and present some mathematical definitions and theorems. Last, we discuss the results and conclude.

2. Models and methods

Consider the following agent-based model. Let $\{1, \cdots, N\}$ be the agents, and at time $t = 0$ we assign a real number $\sigma(i)$ (where $-\infty < \sigma < \infty$) which represents the opinion of agent i about a certain topic of discussion. The agent's opinion can only change due to pairwise interactions between agents engaged in a discussion.

Given the discrete nature of an argument exchange process, we assume that every time two agents interact, they increase or decrease their opinions by a fixed quantity h, which accounts for the influence of the new argument incorporated by the agent. We assume also that both agents are compromising to reach an agreement. So, if agents i and j interact, and $\sigma(i) < \sigma(j)$, then

$$\begin{aligned} \sigma^*(i) &= \sigma(i) + h, \\ \sigma^*(j) &= \sigma(j) - h. \end{aligned} \tag{1}$$

In this way, the persuasion dynamics is not instantaneous and could be interpreted as a discussion process in which agents get closer in opinions with time.

In order to obtain the master equations of this model, let us subdivide the real line in a family of intervals $\{I_j\}_{j\in\mathbb{Z}}$, of length h, and define:

$$s(j,t) = \frac{\#\{i : \sigma(i,t) \in I_j\}}{N}, \tag{2}$$

for $j \in \mathbb{Z}$, as the density of agents with opinion σ in the intervals I_j. Let us note that, being a finite set of agents, we have $s = 0$ outside some interval $[-M, M]$.

Let us deduce the master equation for the density s. Fixing some characteristic time τ related to the rate of interactions, we have

$$s(j, t+\tau) = s(j,t) + \frac{2}{N}(G(j,t) - L(j,t))$$

,where $G(j, t)$ stands for a gain term and $L(j, t)$ for a loss term. In a time interval of length τ only two agents change their opinions, and then the proportion of agents s_j increases or decreases by $1/N$. The factor 2 appears since we can choose an agent located at I_j as the first or the second agent in the interaction.

The gain term G is computed as the probability of an interaction between some agent located at I_{j+1} (respectively, I_{j-1}) at time t and another agent located at I_i with $i \le j$ (resp., $i \ge j$). The loss term L is computed as the probability of an interaction between some agent located at I_j and any another agent outside I_j, since in this case there are no changes.

Therefore, for each $j \in \mathbb{Z}$ we have

$$\begin{aligned} \frac{N}{2}\Big(s(j,t+\tau) - s(j,t)\Big) =& G(j,t) - L(j,t) \\ =& s(j+1,t)\sum_{i\le j} s(i,t) + s(j-1,t) \\ & \sum_{i\ge j} s(i,t) - s(j,t)\sum_{i\ne j} s(i,t) \\ =& \Big(s(j+1,t) - s(j,t)\Big)\sum_{i\le j} s(i,t) - \\ & - \Big(s(j,t) - s(j-1,t)\Big)\sum_{i\ge j} s(i,t) \\ & + 2s^2(j,t), \end{aligned} \tag{3}$$

where we have rearranged the series with the same terms in the last step. Let us recall that this equations must be complemented with the initial distribution at time $t = 0$.

The resulting system of equations is easier to study if considering the continuous version. To this end, we introduce a smooth function $u(x,t) : \mathbb{R} \times [0,\infty) \to \mathbb{R}_{\ge 0}$ such that,

$$s(j,t) = \int_{I_j} u(x,t)dx.$$

This means that u restricted to the interval I_j behaves like $s(j, t)/h$.

Let us observe that, for $x \sim hj$,

$$\begin{aligned} [s(j+1,t) - s(j,t)] =& \frac{h}{h}[s(j+1,t) - s(j,t)] \\ =& h^2\left[\frac{u(x+h,t) - u(x,t)}{h}\right] \\ \approx& h^2\frac{\partial u(x,t)}{\partial x}, \\ \sum_{i\le j} s(i,t) =& \frac{h}{h}\sum_{i\le j} s(i,t) \\ =& \frac{1}{h}\sum_{i\le j} s(i,t)h \\ \approx& \int_{-\infty}^{x} u(y,t)dy, \end{aligned}$$

and therefore

$$\Big(s(j+1,t) - s(j,t)\Big)\sum_{i\le j} s(i,t) \approx h^2\frac{\partial u}{\partial x}\int_{-\infty}^{x} u(y,t)dy.$$

Similar formulas hold for the other differences and sums, so for τ and h small, the equation of the continuous model reads: