Chaos, Solitons and Fractals 97 (2017) 75–83

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

A two-temperature rotating-micropolar thermoelastic medium under influence of magnetic field

Samia M. Said[a,*], Yassmin D. Elmaklizi[b], Mohamed I.A. Othman[a,c]

[a] Department of Mathematics, Faculty of Science, P.O. Box 44519, Zagazig University, Zagazig, Egypt
[b] Department of Mathematics, Faculty of Science, Suez Canal University, Suez, Egypt
[c] Department of Mathematics, Faculty of Science, Taif University 888, Saudi Arabia

ARTICLE INFO

Article history:
Received 15 November 2016
Revised 26 January 2017
Accepted 27 January 2017

Keywords:
Heat source
Magnetic field
Micropolar
Rotation
Two-temperature

ABSTRACT

In the present paper, we introduced a general model of the equations of the formulation in the context of Lord–Shulman theory which includes one thermal relaxation time and Green–Lindsay theory with two thermal relaxation times as well as the classical dynamical coupled theory to study the effect of the rotation and the magnetic field on the total deformation of a micropolar thermoelastic medium with an internal heat source that is moving with a constant speed. The analytical method used to obtain the formula of the physical quantities is the normal mode analysis. Comparisons made with the results of the three theories in the presence and absence of the magnetic field as well as an internal heat source. A comparison is also made with the results of the three theories for different values of the rotation.

List of symbols

σ_{ij}	Stress tensor
μ_0	Magnetic permeability
ε_0	Electric permeability
$\underset{\sim}{J}$	Current density vector
ε_{ijk}	Alternate tensor
m_{ij}	Couple stress tensor
j	Rotational inertia
ρ	Density
E	Dilatation
K	Thermal conductivity
λ, μ	Lame' constants
α_t	Coefficient of linear thermal expansion, $\gamma = (3\lambda + 2\mu)\alpha_t$
t_0, t_1, t_2	Thermal relaxation times
T_0	Reference temperature
δ_{ij}	Kronecker delta
α, β, γ_1, k_0	Micropolar constants
Φ	Conductive temperature
C_E	Specific heat at constant strain
Q	Internal heat source
δ	Two-temperature parameter

1. Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the elastomer and polymer type, e.g. polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves i.e. for the case of elastic vibrations characterized by high frequencies and small wavelengths, influence of the body microstructure becomes significant. This influence of microstructure results in development of new type of waves, not found in the classical theory of elasticity. Metals, polymers, composites, soils, rocks, concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a microstructure. Eringen and Suhubi [1] and Suhubi and Eringen [2] developed the nonlinear theory of micro-elastic solids. Later Eringen [3–5] developed a theory for the special class of micro-elastic materials and called it the “linear theory of micropolar elasticity”. Under this theory, solids can undergo macro-deformations and macro-rotations. Marin and Marinescu [6] discussed asymptotic partition of total energy for the solutions of the mixed initial boundary value problem within the context of

* Coresoponding author.
E-mail addresses: Samia_said59@yahoo.com (S.M. Said), jass.dess@gmail.com (Y.D. Elmaklizi), m_i_a_othman@yahoo.com (M.I.A. Othman).

http://dx.doi.org/10.1016/j.chaos.2017.01.016

the thermoelasticity of initially stressed bodies. Dost and Taborrok [7] presented the generalized thermoelasticity by using Green and Lindsay theory. A domain of influence theorem for microstretch elastic materials was discussed by Marin [8]. Chandrasekharaiah [9] developed a heat flux dependent micropolar thermoelasticity. Marin and Lupu [10] discussed harmonic vibrations in thermoelasticity of micropolar bodies. Kumar [11] investigated the reflection coefficient in micropolar viscoelastic generalized half-space. Kumar and Sharma [12] obtained the amplitude ratios from the stress free boundary in a micropolar thermoelastic half-space without energy dissipation. Sharma and Marin [13] studied reflection and transmission of waves from imperfect boundary between two heat conducting micropolar thermoelastic solids. Othman et al. [14] discussed the effect of initial stress and the gravity field on micropolar thermoelastic solid with microtemperatures.

In classical dynamical coupled theory (CD) of thermoelasticity proposed by Biot [15], the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. The theory of coupled thermoelasticity extended by Lord–Shulman [16] and Green–Lindsay [17] by including thermal relaxation time in constitutive relations. These theories eliminate the paradox of an infinite velocity of heat propagation and termed generalized theories of thermoelasticity. These exist in the following differences between the two theories:

1. Lord–Shulman (L–S) theory involves one thermal relaxation time of thermoelastic process (τ_0).
2. Green and Lindsay theory involve two thermal relaxations time (τ_0, υ_0).
3. L–S energy equation involves first and second time derivatives of strain, whereas the corresponding equation in the G–L theory needs only the first time derivative of strain.
4. In the linear case, according to the approach of G–L theory the heat cannot propagate with a finite speed unless the stresses depend on the temperature velocity, whereas according to L–S theory the heat can propagate with a finite speed even though the stresses there are independent of the temperature velocity.
5. The two theories are structurally different from one another, and one cannot be obtained as a particular case of the other.

Applying the above theories of generalized thermoelasticity, several problems have solved by Ghosh and Kanoria [18], Othman et al. [19]. A theory of the heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature and the thermodynamic temperature, have established by Chen and Gurtin [20] and Chen et al. [21,22]. To time-independent problems, the difference between these two distinct temperatures is proportional to the heat supply and in absence of any heat supply, these two-temperature are identical as Chen et al. [21]. For time-dependent situations and for wave propagation problems, in particular, the two-temperatures are in general different, regardless of presence of a heat supply. Several problems with the two-temperature theory of thermo-elasticity have solved by Warren and Chen [23], Youssef [24], Abbas and Youssef [25], Abbas and Zenkour [26], Zenkour and Abouelregal [27] and Said and Othman [28] etc.

Our main object in writing this paper is to present a micropolar thermoelastic medium with two temperatures in the context of Lord–Shulman (L–S) theory which includes one thermal relaxation time and Green–Lindsay (G–L) theory with two thermal relaxation times as well as the classical dynamical coupled (CD) theory. The governing equations of the problem were solved by using normal mode analysis. The effect of the magnetic field, the rotation, and presence of an internal heat source on the physical quantities studied.

2. Governing equations and formulation of the problem

We consider an isotropic, homengenous and thermoelastic micopolar medium. The elastic medium is rotating with an angular velocity $\boldsymbol{\Omega} = (0, \Omega, 0)$ All quantities will be functions of the time t and of the coordinates x and z. The components of the displacement and the micro-rotation vector will have the form

$$\boldsymbol{u} = (u, 0, w), \quad \boldsymbol{\varphi} = (0, \varphi_2, 0) \tag{1}$$

We consider the elastic medium permeated into a magnetic field of strength $\boldsymbol{H} = (0, H_0 + h, 0)$, where H_0 and $\boldsymbol{h}$ are the initial and induced magnetic field.

The system of governing equations of micropolar thermoelasticity with rotation and Lorentz force is given by [29, 30]:

$$\sigma_{ij,j} + \mu_0(\boldsymbol{J} \wedge \boldsymbol{H})_i = \rho[\ddot{\boldsymbol{u}}_i + \{\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{u}\}_i + (2\boldsymbol{\Omega} + \dot{\boldsymbol{u}})_i] \tag{2}$$

$$\varepsilon_{ijk}\sigma_{jk} + m_{ji,j} = \rho j[\ddot{\varphi}_i + (\boldsymbol{\Omega} \wedge \dot{\boldsymbol{\varphi}})_i] \tag{3}$$

The constitutive laws are

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} + k_0(u_{j,i} - \varphi_r\varepsilon_{ijr}) - \gamma\left(1 + t_1\frac{\partial}{\partial t}\right)(T - T_0)\delta_{ij} \tag{4}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma_1\varphi_{j,i} \tag{5}$$

The strains can be expressed in terms of the displacement u_i as

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad e_{kk} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \tag{6}$$

In the above equations, a comma followed by a suffix denotes partial derivative with respect to the corresponding coordinates.

The generalized heat conduction equation as [31]

$$K\Phi_{,ii} = \rho C_E(t_0 + t_2)\ddot{T} + \rho C_E\dot{T} + \gamma T_0\left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)e - \left(1 + t_0\frac{\partial}{\partial t}\right)Q \tag{7}$$

The relation between the conductive temperature and the thermodynamic temperature is given by

$$\Phi - T = \delta\Phi_{,ii} \tag{8}$$

Due to application of initial magnetic field H_0, there are results of an induced electric field $\boldsymbol{E}$. The linearized equations of electromagnetism of a slowly moving medium are [28]

$$\nabla \wedge \boldsymbol{H} = \boldsymbol{J} + \varepsilon_0\frac{\partial \boldsymbol{E}}{\partial t}, \quad \nabla \wedge \boldsymbol{E} = -\mu_0\frac{\partial \boldsymbol{H}}{\partial t}, \quad \nabla \cdot \boldsymbol{H} = 0, \quad \boldsymbol{E} = -\mu_0\dot{\boldsymbol{u}} \wedge \boldsymbol{H} \tag{9}$$

Expressing the components of the current density vector $\boldsymbol{J}$ in terms of displacement by eliminating the quantities $\boldsymbol{E}$ and $\boldsymbol{H}$ from Eq. (9), we get

$$\boldsymbol{J} = \left(-\frac{\partial h}{\partial z} - \mu_0\varepsilon_0 H_0\frac{\partial^2 w}{\partial t^2},\ 0,\ \frac{\partial h}{\partial x} + \mu_0\varepsilon_0 H_0\ \frac{\partial^2 u}{\partial t^2}\right) \tag{10}$$

Eqs. (2) and (7) are the field equations of the generalized thermoelasticity of elastic solid, applicable to the classical coupled theory CD, L–S theory as well as G–L theory as follows [32]:

(a) Equations of the coupled theory (CD), when $t_1 = t_2 = t_0 = 0$.
(b) Lord–Shulman (L–S) theory, when $t_0 \neq 0$, $t_1 = t_2 = 0$.
(c) Green–Lindsay (G–L) theory, when t_1, $t_2 \neq 0$, $t_0 = 0$.
(d) The corresponding equations for the three theories without magnetic field from the above mentioned cases by taking $H_0 = 0$.
(e) The corresponding equations for the three theories without rotation from the above mentioned cases by taking $\Omega = 0$.