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Analysis of spatial chaos appearance in cascade connected nonlinear electrical circuits

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ABSTRACT

The system consisting of several cascade connected electrical circuits is presented in this paper. Considering the system structure and the fact that the tunnel diodes have nonlinear characteristics, one of the properties of this system is the possibility of the chaos appearance. Necessary conditions and sufficient condition for the chaos appearance in the nonlinear cascade connected systems are given and analyzed, too. The results are confirmed by bifurcation and escape-time diagrams simulation.

1. Introduction

It was noticed, in the last years that in some practical realization of cascade connected nonlinear systems, (for example, at systems consisting of several cascade connected transporters for transportation of plastic or rubber strip materials, [1,2]) very complex oscillations appear. Under large amplifications these oscillations become complex motions which cannot be described in the classic manner, [3–6]. It was shown that these motions present a type of deterministic chaos and that bifurcations can appear when the control parameter of cascade system varies. Particularly, in this case chaos appearance is not the result of signal iteration through the time, but the signal running through the space, [1,2,7–10].

Mathematical model of these cascade connected nonlinear systems is described as follows:

$$x_{k+1} = f(x_k,\ r) \tag{1}$$

where x_k is the input, x_{k+1} is the output, r is the amplification of the kth cascade and f is the nonlinear function which is the same for all cascades.

In the Fig. 1 the block scheme of nonlinear cascade system, given by Eq. (1), is shown. With x_1 the input of the first cascade is marked, $f = f(x_i, r), i = \overline{1, k}$ is the two argument nonlinear function, x_i is the output of the previous cascade and r is the amplification which is the same for all cascades. Therefore, the output of the ith cascade is at the same time the input of the next $(i+1)$th cascade.

In order to have a chaos appearance in electrical circuits, there must be a nonlinear element in the circuit, i.e., the element with nonlinear current-voltage characteristic, for example nonlinear resistor, diode, etc. The simple examples of electric circuits where chaos appears are Van der Pol oscillator, and Chua's circuit, [6]. Both circuits have one nonlinear element, i.e. nonlinear resistor.

Professor L. Chua was the first who proved mathematically the existence of deterministic chaos in the system of differential equations. He, also, proved obtained result using the computer and, at the end, by experiment on the real system. This real system called Chua's circuit is described by the initial system of differential equations. For the different shapes of the current-voltage characteristic of nonlinear resistor and different values of $C_1,\ C_2,\ R$ and L, Chua's circuit has very rich dynamic behavior in parametric plane $(\alpha,\ \beta)$ where $\alpha = C_2/C_1,\ \ \beta = C_2R^2/L$. For $\beta =$ const. and for different values of parameter α, Hopf bifurcation, Rossler attractor etc., appear in circuit. (Figs. 2 and 3)

In order to have a chaos appearance in cascade connected nonlinear electrical circuit it is necessary that two conditions are satisfied:

1 each cascade has one nonlinear element,
2 excitation is in the first cascade and the output of one cascade is the input of the next cascade.

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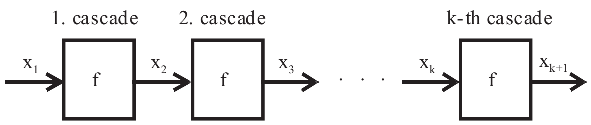

Fig. 1. Block scheme of nonlinear cascade system.

The system consisting of several cascade connected nonlinear electrical circuits is presented in this paper. Every cascade has one nonlinear element, tunnel diode. The necessary conditions and sufficient condition for the chaos appearance in mentioned system is analyzed using Matlab. The results are illustrated using the Matlab M-files for obtaining the bifurcation diagram and the escape-time diagram.

Let the Eq. (1) describes nonlinear cascade system in the steady state. Bifurcation diagram of this cascade system shows outputs of all cascades in steady state for different values of control parameter, i.e., amplification r. In this case, parametric axis is divided into the set of close points and, unlike the nonlinear discrete systems, the Eq. (1) is iterated only once for each cascade in every point of parameter r.

An escape-time diagram presents the procedure for the examination of chaotic motion of system (1) and visually shows the degree of expansion for each point in the examined region. For example, each point with coordinates (r, x_1), (x_1 is input in the first cascade, i.e., the input of nonlinear cascade system) in Decart plane, can be colored differently (or colored in a shade of the same color). The point color is determined depending on iteration number needed for point to reach the, in advance, given value of cascade output. Since, only eight colors are available, points corresponding to larger number of iterations, are colored in the same color as points corresponding to smaller number of iterations. For example, the point with nine iteration is colored the same as the point with one iteration, the point with ten iterations is colored the same as the point with two iterations, etc.

There are some new methods in nonlinear circuits for improving precision in chaotic system analysis, such as perturbing chaotic states and cascading multiple chaotic systems [11]. Also, analysis of chaos is especially beneficial to practical applications in communication, [12,13].

2. The necessary and sufficient conditions for chaos appearance in cascade connected nonlinear systems

Chaotic motion was initially analyzed theoretically for discrete nonlinear systems, i.e., for iterative processes described by Eq. (1), where r represents variable parameter, x_k is a state vector, $f$is nonlinear function, and k is discrete time, [3–6]. So, in this case iteration is running through time. Bifurcation occurs at certain value of parameter r, and if its value is increased enough, depending on function f, a chaotic motion may appear.

Deterministic chaos theory, applied to discrete nonlinear systems (1), can be applied to cascade connected nonlinear systems, also. In that case, iteration is not running through time, but rather in space. Each cascade in the system represents one iteration, i.e., k in Eq. (1) in not discrete time, but represents cascade serial number. While in the case of discrete nonlinear systems (1) iteration occurs in the same system during the time, in cascade systems a signal passing through the series of cascade connected nonlinear subsystems of the same structure represents iterative process. Mapping the time chaos to the spatial chaos, i.e., the fact that the same theory developed for discrete nonlinear systems (1) can be applied for the analysis of cascade connected nonlinear systems is presented in this paper.

Let S_1 denotes the set of points obtained by mapping $y = f(x_1, r)$ for all possible positive input values to the first cascade x_1, when r is constant parameter, i.e.:

$$S_1 \ : \ y = f(x_1, r), \ x_1 > 0, \ r = \text{const.} \tag{2}$$

Set S_1 represents the set of all possible outputs of the first cascade, x_2, when r is constant parameter.

S_0 is the set of points of the line $y = x_1$, i.e.:

$$S_0 \ : \ y = x_1, \ x_1 > 0. \tag{3}$$

Hence, set S_0 represents a set of all possible values of the inputs to the first cascade.

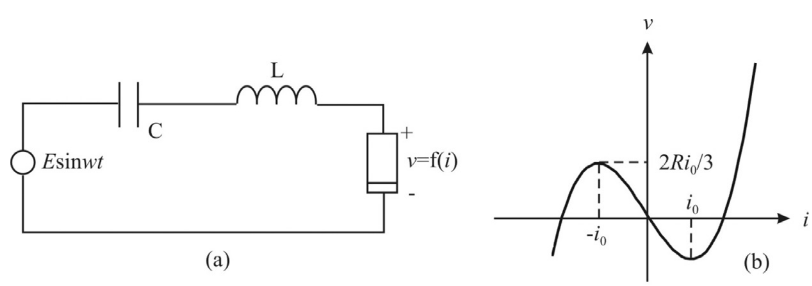

Fig. 2. a) Van der Pol oscillator, b) Voltage-current characteristic of nonlinear resistor in the circuit of Van der Pol oscillator.

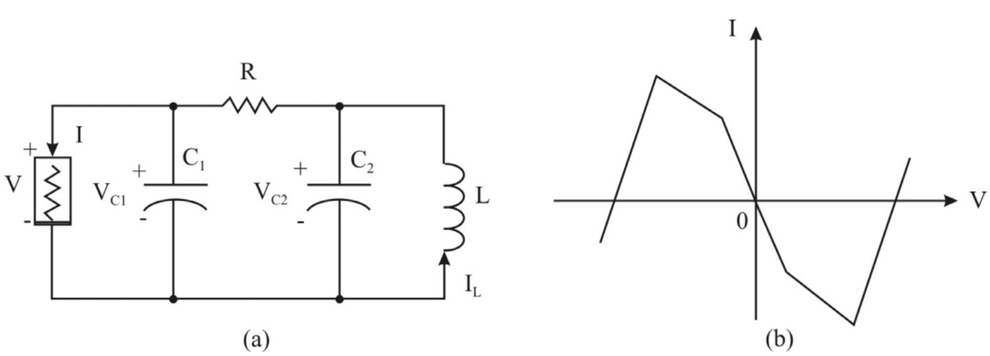

Fig. 3. a) Chua's circuit, b) Current-voltage characteristic of nonlinear resistor in the Chua's circuit.