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Risk preference, option pricing and portfolio hedging with proportional transaction costs☆

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ABSTRACT

This paper is concerned in the option pricing and portfolio hedging in a discrete time case with the proportional transaction costs. Through the Monte Carlo simulations it has been shown that the fractal scaling and risk preference of traders have an important influence on the hedging performances in both option pricing and portfolio hedging in a discrete time case. In addition, the relation between preference of traders and implied volatility frown is discussed. We conclude that the risk preferences of traders play an important role in determining the shape of the implied-volatility-frown and the different options having the different hedging frequencies is another reason for the implied volatility frown.

1. Introduction

Since the publication of the works of Black and Scholes [1] and Merton [2], the interest in option pricing has dramatically increased. An essential feature of the Black and Scholes as well as Merton approaches is that the trading is assumed to make in a continuous-time manner so that the price of any option does not depend on the time scaling and traders' risk preferences (scaling-free pricing and preference-free pricing). However, in recent years many researchers have discovered that a number of financial market data display some complex and nonlinear characters. A series of studies have found that many financial market time series exist the scaling law [3–13]. Therefore, it has been suggested that one should consider the influence of the scaling and preference on option pricing and portfolio hedging. In this paper, through the Monte Carlo simulations for the independently 1000 sample paths we show that the risk preference of traders and the fractal scaling [13] as well as the proportional transaction costs play an important role in option pricing and portfolio hedging.

The problem of the option pricing and the portfolio hedging in a discrete time case with the proportional transaction costs has been studied by many authors starting with Leland [14], Boyle and Emanuel [15], Lott [16], Wilmott [17,18], up to more recent works [19–29]. All those authors [13–29] show that the fractal scaling of traders has an important influence on option pricing and portfolio hedging, but they did not consider the effect of the risk preferences of traders on the hedging performances in both implied-volatility-frown and option pricing. In fact, while there exist the proportional transaction costs the markets are incomplete. In those cases, the option prices are heavily dependent on the risk preferences of the traders. In the mean time, many econophysicists are also interested in analyzing financial time series through using different fractal scaling δt to research the complex structures of economic systems. In particular, Mantegna and Stanley [5,6], Stanley and Plerou [7] and Stanley et al. [8] introduced the method of scaling invariance from complex science into economic systems for the first time. Since then, many researches on scaling laws in econophysics have taken place. Mandelbrot [3,4] and Mantegna and Stanley et al. [5–8] considered the problem of choosing the appropriate fractal scaling to analyses financial market data and price options. Bouchaud and Potters [9] and Potters et al. [10]. introduced an asymptotic method to tackle the residual risk and proposed to find the optimal strategies to price options. In this paper, on basis of the points of view of behavioral finance [30, 31] and econophysics [3–10,32] we will use the mixed hedging strategy [33] to price the options while there exist the proportional transaction costs.

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This paper is organized as follows. In Section 2, through the mixed hedging strategy $X_1(t)$, a new option pricing formula is obtained with the proportional transaction costs and we show that the proportional transaction costs and the fractal scaling as well as the risk preference play an important role in option pricing. In Section 3, the Monte Carlo simulations for the independently 1000 sample paths are given to show that the mixed hedging strategy $X_1(t)$ is an improvement over the Leland hedging and the modified Leland hedging in high-frequency trading in the "real world" as $X < S_0$, even if this evidence is not absolutely conclusive. In Section 4, the relation between the risk preference of traders and the implied volatility frown is discussed. We conclude that the risk preferences of traders play an important role in determining the shape of the implied-volatility-frown and the different options having the different hedging frequencies is another reason for the implied volatility frown. Section 5 concludes.

2. Option pricing with proportional transaction costs

In this section a new option pricing formula will be obtained while there are the proportional transaction costs. We show that the proportional transaction cost parameter k and the fractal scaling δt as well as the risk preference parameter μ play an important role in option pricing while the continuous time trading assumption is given up.

Leland [14] has derived a simple model for pricing options in the presence of transaction costs. He adopted the delta hedging strategy of rehedging at every time interval δt. That is, every δt the portfolio is rebalanced, whether or not the asymptotic replication error tends to zero in probability. In the proportional transaction cost option pricing model, we follow the other usual assumptions in the Black–Scholes model but with the following exception.

(i) The portfolio is revised every δt, where δt is a finite and fixed, small time interval.

(ii) Transaction costs are proportional to the value of the transaction in the underlying. Let k denote the round trip transaction cost per unit dollar of transaction. Suppose υ shares are bought ($\upsilon > 0$) or sold ($\upsilon < 0$) at the price S, then the transaction cost is given by $\frac{k}{2}|\upsilon|S$ in either buying or selling. The value of the constant k will depend on the individual investor.

(iii) The hedged portfolio has an expected return equal to that from an option. This is exactly the same valuation policy as earlier on discrete hedging with no transaction costs.

(iv) In the paper [33], in order to show that the residual risk and trade scaling play an important role in the Black–Scholes option pricing model, a mixed hedging strategy $X_1(t)$, i.e.,

$$X_1(t) = \frac{\partial C}{\partial S_t} + \frac{\mu\delta t}{1+\mu\delta t}\frac{\partial^2 C}{\partial {S_t}^2}S_t \tag{2.1}$$

has been proposed to price options in a frictionless financial market. Now we assume that traders make use of the mixed hedging strategy $X_1(t)$ to price options while there exist proportional transaction costs.

(v) Empirical findings [34,35] show that the price of a European option is a convex function of the underlying stock price. Therefore, we assume that $\frac{\partial C}{\partial S_t}>0$, and $\frac{\partial^2 C}{\partial {S_t}^2}>0$.

In addition, in our model where transaction costs are incurred at every time the stock or the bond is traded, the no arbitrage argument used by Black and Scholes no longer applies. The problem is that due to the infinite variation of the geometric Brownian motion, perfect replication incurs an infinite amount of transaction costs.

Now consider a simple financial market model with constant coefficients, which consists of a stock and a bond with price dynamics given by

$$S_t = S_0 e^{\mu t+\sigma B_t}, \tag{2.2}$$

and

$$D_t = D_0 e^{rt}, \tag{2.3}$$

where μ, $\sigma \neq 0$, $S_0 > 0$, $r > 0$, $t \in [0, T]$, $T \in R$ fixed, and $\{B_t\}_{t\in[0,T]}$ a standard one dimensional Brownian motion on a complete probability space (Ω, F_t, P) which is equipped with the $P-$augmentation $\{F_t\}_{t\in[0,T]}$ of the natural Brownian filtration.

After a small time interval δt, the price changes in the bond and in the stock are

$$\delta D_t = rD_t\delta t + O\big((\delta t)^2\big), \tag{2.4}$$

$$\begin{aligned}\delta S_t &= S_t\left[e^{\mu\delta t+\sigma\delta B_t} - 1\right]\\ &= S_t\left[\mu\delta t + \sigma\delta B_t + \frac{\sigma^2}{2}(\delta B_t)^2\right] + G_1(\delta t),\end{aligned} \tag{2.5}$$

$$E[\delta S_t] = S_t[\mu\delta t + \frac{\sigma^2\delta t}{2}] + E[G_1(\delta t)], \tag{2.6}$$

and

$$E[(\delta S_t)^2] = S_t^2\sigma^2\delta t + E[G_2(\delta t)], \tag{2.7}$$

where

$$E[G_i(\delta t)] = O\big((\delta t)^2\big) \quad i = 1, 2. \tag{2.8}$$

Let $C=C(t, S_t)$ be the value of a European call on the above underlying stock at time t with expiration date $T$and exercise price X and the boundary conditions:

$$C(T, S_T) = (S_T - X)^+ \quad \text{at } t = T, \tag{2.9}$$

and

$$C(t, 0) = 0,$$

where $C(t, S_t)$ is assumed to have continuous partial derivatives up to order three.

Consider a replicating portfolio Π_t with $X_1(t)$ units of the stock and $X_2(t)$ units of the bond. The value of the portfolio is given by

$$\Pi_t = X_1(t)S_t + X_2(t)D_t, \tag{2.10}$$

After the time interval δt, the change in the value of the portfolio is

$$\delta\Pi_t = X_1(t)\delta S_t + X_2(t)\delta D_t - \frac{k}{2}|\delta X_1(t)|S_{t+\delta t}, \tag{2.11}$$

Since $C(t, S_t)$ is assumed to have continuous partial derivatives up to order three, the change in the value of the option is

$$\delta C = \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial S_t}\delta S_t + \frac{1}{2}\frac{\partial^2 C}{\partial {S_t}^2}(\delta S_t)^2 + G_3(\delta t), \tag{2.12}$$

where

$$E[G_3(\delta t)] = o(\delta t) \tag{2.13}$$

Similar to Leland's argument [14], if δt is sufficiently small, from Eq. (2.1) we have

$$\begin{aligned}\delta X_1(t, S_t) = &\frac{\partial X_1(t, S_t)}{\partial S_t}\delta S_t + \frac{\partial X_1(t, S_t)}{\partial t}\delta t\\ &+ \frac{1}{2}\frac{\partial^2 X_1(t, S_t)}{\partial S_t^2}(\delta S_t)^2 + G_4(\delta t)\end{aligned}$$