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Seasonality and the logistic map

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ABSTRACT

Nonlinear difference equations, such as the logistic map, have been used to study chaos and also to model population dynamics. Here we propose a model that extends the “lose + lose = win” behavior found in Parrondo’s Paradox, where switching between chaotic parameters in the logistic map yields periodic behavior (“chaos + chaos = order”). The model uses twelve parameters each reflecting the conditions of one of the twelve months. In this paper we study the effects of smooth-transitioning parameters and the robust system that emerges.

1. Introduction

Nonlinear difference equations, such as the logistic map, have been studied as a way to model ecological systems [1]. Populations with growth over discrete time intervals and nonoverlapping generations can be modeled with the logistic map [2]. Switching strategies with parameters in the logistic map can result in dynamics that reflect a behavior similar to the “lose + lose = win” found in Parrondo’s Paradox. Parrondo’s Paradox in game theory consists of two games that independently each result in loss, are alternated to result in a winning game [3]. The apparent paradox is a result of the combination of random behavior with an asymmetry [4]. The “lose + lose = win” dynamics of Parrondo’s Paradox have been alluded to “chaos + chaos = order” dynamics found in the switching of two chaotic parameters in logistic maps [5,6]. In this example two parameters that alone each result in chaos can yield, if switched, periodic behavior.

Previously, we have extended the two parameter model to a four parameter system to reflect the four seasons with specific parameters chosen to reflect the conditions of each of the four seasons [7]. Switching between the four parameters, allow us to study its periodic behavior that could be used to model population dynamics. This model combines conditions that individually would be undesirable, but together yield ordered, desirable dynamics. Here we extend the four season model to a twelve month model. We choose twelve parameters between 0 (extinction) and 4 (chaos) depending on the typical weather conditions for each month. Kot and Schaffer studied a two parameter seasonality model and found that mild seasonality was stabilizing for a system, but large seasonality inevitably lead to chaos [8]. With our twelve parameter model we show that even a large range of seasonality can yield stable, periodic behavior when the transitions are gradual.

In Section 2 we first lay out the logistic map and the results from switching between two parameters, and we briefly review the four parameter seasonality model. In Section 3 we introduce the twelve month switching model and explain why we chose certain parameters to represent certain months. We consider bifurcation diagrams and periodic trajectories for our model and explain the resulting observations about the stabilizing effects of smooth seasonality. We close in Section 4 with a discussion of our results.

2. Switching in the logistic map

In this section, we start with the definition of the logistic map:

$$X_{n+1} = cX_n(1 - X_n) \tag{1}$$

where X_n is less than unity and c lies in the interval [0,4].

In Fig. 1 we show the bifurcation diagram found by removing transient iterations and plotting the final 200 values of X_n. As it has been reported by many researchers, we notice that the diagram bifurcates at $c = 3$, and then breaks into chaos at about $c = 3.569$. Now we introduce the map with two alternating parameters of c_n, where we designate c_o for odd n and c_e, for even n. Furthermore we can set one of the $c's$ to a fixed value and study the bifurcation diagrams yielded when we vary the second c parameter.

$$X_{n+1} = \begin{cases} c_e X_n(1 - X_n) & \text{if } n \text{ even,} \\ c_o X_n(1 - X_n) & \text{if } n \text{ odd.} \end{cases} \tag{2}$$

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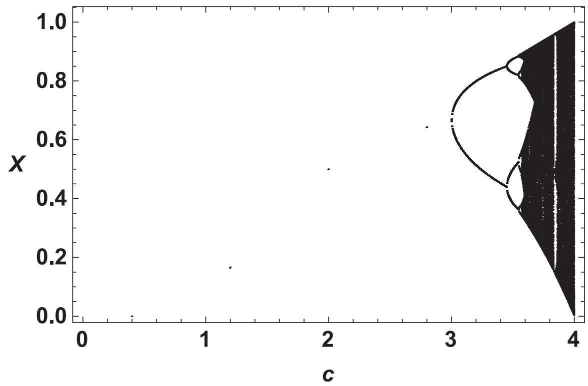


Fig. 1. For the bifurcation diagram of the logistic map, Eq. (1), we plot 200 iteration after discarding transient iterations.

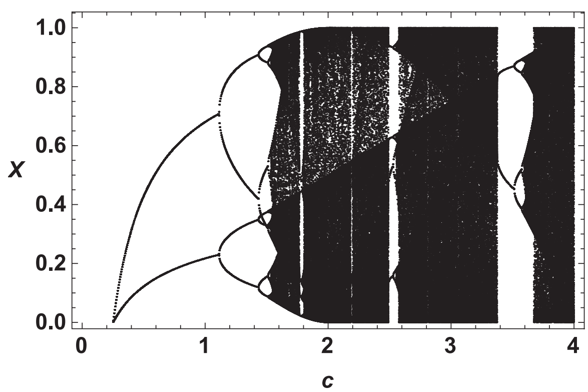


Fig. 2. For the bifurcation diagram of the switched logistic map, Eq. (2), with $c_o = 4$ and $c_e = c$, we plot 200 iterations after discarding transient iterations.

In Fig. 2 we plot the logistic map with alternating values of c_e with $c_o = 4$, a chaotic parameter. In this case, we observe that for many values of c_e, we can find regions of periodicity. Moreover we can chose $c_e = 3.634$, which lies in the chaotic region of the simple logistic map, and find oscillations of period two, as shown in Fig. 3. Independently each parameter leads to chaos, but switching between them gives periodicity, thus creating a situation where chaos can be controlled. This is an example of Parrondo-like "undesirable + undesirable = desirable" behavior. In particular, in Fig. 3, we depict the periodic orbit that emerges from alternating two "chaotic" parameter values.

In previous work we have extended the two parameter switching to the four parameter switching modeling seasonality. We chose two small parameter less than unity that would lead a population to extinction (winter and fall) and two larger parameter values that would lead a population to chaos (spring and summer). Both types of conditions alone are undesirable, but cycling through them can give regular patterns. This approach can help model population dynamics as a response to a changing environment and how the population may stay stable even under general undesirable conditions.

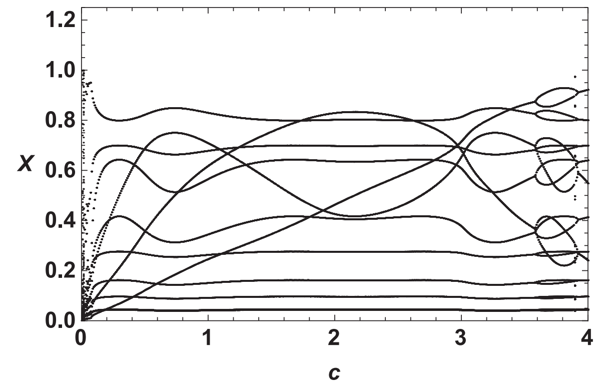


Fig. 4. Bifurcation diagram for Eq. (3), where $c_7 = c$ is the bifurcation parameter and other parameter values taken from Table 1.

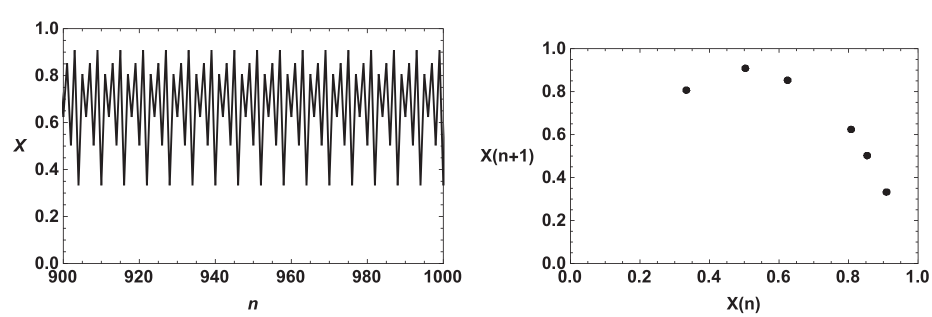
3. Twelve month switching model

As an extension of the study of seasonality using switching strategies in the logistic map, here we consider twelve parameters modeling the twelve months of the year. We cycle through twelve units in time, each value of c representing a month, with the following logistic equation:

$$X_{n+1} = \begin{cases} c_0 X_n(1-X_n) & \text{if mod}[n,P] = 0, \\ c_1 X_n(1-X_n) & \text{if mod}[n,P] = 1, \\ c_2 X_n(1-X_n) & \text{if mod}[n,P] = 2, \\ c_3 X_n(1-X_n) & \text{if mod}[n,P] = 3, \\ c_4 X_n(1-X_n) & \text{if mod}[n,P] = 4, \\ c_5 X_n(1-X_n) & \text{if mod}[n,P] = 5, \\ c_6 X_n(1-X_n) & \text{if mod}[n,P] = 6, \\ c_7 X_n(1-X_n) & \text{if mod}[n,P] = 7, \\ c_8 X_n(1-X_n) & \text{if mod}[n,P] = 8, \\ c_9 X_n(1-X_n) & \text{if mod}[n,P] = 9, \\ c_{10} X_n(1-X_n) & \text{if mod}[n,P] = 10, \\ c_{11} X_n(1-X_n) & \text{if mod}[n,P] = 11. \end{cases} \quad (3)$$

where $P = 12$.

In this model, we choose three parameter values between 0 and 1, modeling conditions leading to extinction, for the three coldest months of the year, December, January, and February. Next we choose three values between 1 and 3, which yield steady



(a) Time series (b) Return map

Fig. 3. Time series and return map for Eq. (2), with $c_o = 4$ and $c_e = 3.634$.