Chaos, Solitons and Fractals 93 (2016) 111–122

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Ecological dynamics of age selective harvesting of fish population: Maximum sustainable yield and its control strategy

Debaldev Jana[a], Rashmi Agrawal[b], Ranjit Kumar Upadhyay[b,*], G.P. Samanta[c]

[a] Department of Mathematics, SRM University, Kattankulathur, Tamil Nadu-603 203, India
[b] Department of Applied Mathematics, Indian Institute of Technology (Indian School of Mines), Dhanbad - 826 004, Jharkhand, India
[c] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, India

ARTICLE INFO

Article history:
Received 18 July 2016
Revised 21 September 2016
Accepted 23 September 2016

Keywords:
Age selective harvesting
Maximum sustainable yield
Bionomic equilibrium
Optimal harvesting

ABSTRACT

Life history of ecological resource management and empirical studies are increasingly documenting the impact of selective harvesting process on the evolutionary stable strategy of both aquatic and terrestrial ecosystems. In the present study, the interaction between population and their independent and combined selective harvesting are framed by a multi-delayed prey-predator system. Depending upon the age selection strategy, system experiences stable coexistence to oscillatory mode and vice versa via Hopf-bifurcation. Economic evolution of the system which is mainly featured by maximum sustainable yield (MSY), bionomic equilibrium and optimal harvesting vary largely with the commensurate age selections of both population because equilibrium population abundance becomes age-selection dependent. Our study indicates that balance between harvesting delays and harvesting intensities should be maintained for better ecosystem management. Numerical examples support the analytical findings.

1. Introduction

Age or body size is generally considered to be one of the most important and significant traits of a species because it correlates with many aspects of its biology, from life history to ecology [7,15,36,50]. The views of fisheries management over the last few decades, have oscillated between alarm and trust in management progress. The predominant scientific policies for remedying the world fishing crisis aim at maximum sustainable yield (MSY) by selective harvesting which is by adjusting gear selectivity, fishing effort etc. Law and Grey [30] by a theoretical work on north-east Arctic cod pointed out the concept of fisheries-induced evolution and its effects on the yield of an exploited fish stock. In this study they show how fishing pressure creates a strong selection for individuals with early maturation, leading to a change in the age at maturation of the stock, towards earlier maturation. Correlated life-history traits in response to size-selective harvesting have been demonstrated in laboratory populations of the Atlantic silverside, *Menidia menidia* [10,52].

Different case studies on Hilsa fish at Hooghly estuary say a real scenario of ecological and economical status of India and Bangladesh. De and Datta [12] did an experiment of Indian shad, *Tenualosa ilisha* (Hamilton) commonly known as Hilsa, of the freshwater zone of Hooghly estuary by using the length- frequency method. They used different techniques e.g. Von Bertalanffy growth equation to estimate the length-weight relationship of Hilsa fish. It is well known that the rate of growth of fish population may vary from one environment to other or in the same environment from year to year due to changes in ecology including changes in food availability, density dependent growth factors etc [38,22,43,39,44,42]. The low yield of Hilsa in present day situation has arrived a question to find out the factors like indiscriminate killing of juveniles, establishment of Farakha Barrage, decreasing depth of estuary, pollution state [3]. Also, it is observed by the empirical data on Serampore-Uttarpara belt [3] that there is a very important biological as well as economic impact of length-weight relationship of Hilsa fish on fishery management. Over fishing and over exploitation of juveniles play a very negative role in any economically profitable fishery which in the broad sense stress on the biological and economic equilibrium and that also emphasize on the evolutionary consequences of the population and its environment. Gear selection for particular size of fish [3], perfect boat choice [20], government policies to prevent juvenile fish and protect overfishing [3,20] etc are the prominent solutions and real application to prevent ecosystem management and biodiversity.

Presently, management objectives emphasize not only the biological yield of fisheries (MSY) but also include economic and social considerations. Many coastal rural communities are solely de-

* Corresponding author. Fax: +913262296559.
E-mail address: ranjit.chaos@gmail.com (R.K. Upadhyay).

http://dx.doi.org/10.1016/j.chaos.2016.09.021
0960-0779/

pendent on particular fisheries [3,22,38,39,42–44]. In most cases, fisheries management is conducted for regulating fishing mortality [20,46]. Methods to regulate fishing mortality include direct methods such as size and catch limits and indirect methods such as seasonal closures, area closures, gear restrictions and licensing [3]. In direct methods, the most important is catch limits (number of tons per year). Among the indirect methods, the amount of time that fishermen spends to harvest a species is reduced by seasonal closing of fishery which actually reduces the total harvest. But this method has some drawbacks, sometimes it may encourage fishermen to give more effort during the fishing season and virtually the total harvest is not reduced. For the purpose of economically profitable as well as biologically beneficial system, bionomic equilibrium gives us constraints to have profitable harvesting [31,17,18]. For the interest of getting MSY and bionomic equilibrium of a profitable fishery, optimization is a technique which maximizes the profit without any harmful effect on the system. Optimal harvesting policy gives us a strategy which helps us in both maximizing the profit by minimum level of effort as well as eradicating the risk of extinction of the harvested population [27,14].

Proper harvesting or age/ size selective harvesting is a very significant methodology that dampen fluctuations by making overcompensatory dynamics under compensatory [6,8,9,25,33]. Combined effect of harvesting and delay on prey-predator system are studied by several researchers [5,32,53,54]. Recently, Jana et al. [21] has been established a age-selective harvesting model by using the technique of Arino et al. [2]. For this consequence in the present work, we consider two delays: one in the fish prey harvesting term and the other in fish predator harvesting term. Thus, we are considering selective harvesting of both groups species. Biologically, it implies that we are restricting harvesting of species below some certain age so that they can grow up to some specific size or age and thus protecting juvenile fish population. According to new paradigm of ecosystem based fisheries management (EBFM), selective fishing is widely encouraged in contrary to nonselective fishing which has many adverse impacts [37,41]. Zhou et al. [56] proposed a balanced exploitation approach and commented that selective harvesting should be employed with reduced exploitation rates.

The objective of this article is to study the effect of selective harvesting of either or both species on the dynamics of the system. It is also our intention to verify rationality of some of the above observations on EBFM. The organization of the paper is as follows: In Section 2, we formulate the mathematical model. Also, boundedness, equilibria analysis and existence of Hopf-bifurcation for different cases are derived in this Sect. In Section 3, we present the sustainable yield and MSY policy of the harvested population. Bionomic equilibrium of the model is presented in Section 4. In Section 5, we have presented optimal harvesting policy of the model system. Finally, a brief discussion has been drown from the manuscript in Section 6.

2. Formulation of mathematical model and its analysis

The Rosenweig-McArthur type prey-predator system was studied by Kuang and Freedman [29] which shows an unique limit cycle in the positive quadrant. The case of instantaneous harvesting of both the population has been studied by Srinivasu et al. [49] through the following model:

$$\frac{dx}{dt} = \alpha x\left(1-\frac{x}{k}\right) - \frac{\beta xy}{1+ax} - E_1 x,$$
$$\frac{dy}{dt} = \frac{c\beta xy}{1+ax} - \gamma y - E_2 y, \tag{1}$$

where x is the logistically growing prey population density with intrinsic growth rate α to its environmental carrying capacity k and y is the predator population density. The predation process obeys the Holling type II functional response with attack rate β, attack rate multiplied by handling time is a, food conversion efficiency c and food independent mortality rate γ. Also, it is assumed that both prey and predator are harvested instantaneously by the rates E_1 and E_2 respectively.

It is well known that the rate of change of the population depends on three components: growth, death, and intraspecific competition (crowding or direct interference). The death and the intraspecific competition rates together known as the decline rate. We assume that the decline rate is instantaneous and that the death rate is given by a linear term whereas the intraspecific competition rate is given by the quadratic term. For this, we can write the classical logistic ODE model [2] as follows:

$$\frac{dN(t)}{dt} = bN(t) - \mu N(t) - \delta N^2(t), \tag{2}$$

where b, μ and δ are the birth, death and death due to intraspecific competition respectively of the population $N(t)$ and we choose $\alpha = b - \mu$ (i.e. intrinsic growth rate of $N(t)$). In this equation, the decline rate is given by $\{\mu N(t) + \delta N^2(t)\}$ and the growth rate is given by $bN(t)$. Both are assumed to depend on the present population size. We continue to assume that the decline rate is instantaneous, but we now assume that the growth rate is proportional to the number of individuals in the population $(t-\tau)$ time units in the past, that manage to survive until time t [2].

To obtain an equation that describes how many individuals alive at time $(t-\tau)$ are still alive at time t, we solve the following first order ODE for $N(t)$ as a function of $N(t-\tau)$:

$$\frac{dN(t)}{dt} = -\mu N(t) - \delta N^2(t). \tag{3}$$

Using the technique of separation of variables and integrating both sides from $(t-\tau)$ to t, we obtain [2]

$$N(t) = \frac{\mu N(t-\tau)}{\mu e^{\mu\tau} + \delta(e^{\mu\tau}-1)N(t-\tau)}. \tag{4}$$

Now, if the population does not posses the death due to intraspecific competition (i.e. $\delta = 0$), then the survival rate of population $N(t)$ is given by

$$N(t) = e^{-\mu\tau}N(t-\tau). \tag{5}$$

Since we assume that growth in the population at time t is proportional to those individuals alive at time $(t-\tau)$ who survive until time t, we replace x and y in the monomial with coefficient E_1 and E_2 respectively in (1) by (4) and (5). According to the assumption of selective harvesting, our model system (1) becomes

$$\frac{dx}{dt} = \alpha x\left(1-\frac{x}{k}\right) - \frac{\beta xy}{1+ax} - \frac{E_1\mu x(t-\tau_1)}{\mu e^{\mu\tau_1} + \delta(e^{\mu\tau_1}-1)x(t-\tau_1)},$$
$$\frac{dy}{dt} = \frac{c\beta xy}{1+ax} - \gamma y - E_2 e^{-\gamma\tau_2}y(t-\tau_2), \tag{6}$$

where $\alpha = b - \mu$ (b and μ are linear birth and death rate of prey(x) population), $\delta = \frac{\alpha}{k}$. τ_i $(i = 1, 2)$ are the delays regarding age selection for harvesting of x and y population respectively. The initial conditions of (6) are given as

$$x(\theta) = \phi_1(\theta) \geq 0, y(\theta) = \phi_2(\theta) \geq 0,$$

$$\theta \in [-\tau, 0], \quad \phi_i(0) > 0 \quad (i = 1, 2),$$

where $\phi : [-\tau, 0] \to \Re^2$ are continuous and $\tau = max[\tau_1, \tau_2]$ with norm

$$||\phi|| = \sup_{-\tau\leq\theta\leq 0}\{|\phi_1(\theta)|, |\phi_2(\theta)|\},$$

such that $\phi = (\phi_1, \phi_2)$.