Chaos, Solitons and Fractals 93 (2016) 130–135

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

An improved public goods game model with reputation effect on the spatial lattices

Tianwei Zhou [a,b], Shuai Ding [a,b,*], Wenjuan Fan [a,b], Hao Wang [a,b]

[a] School of Management, Hefei University of Technology, Hefei 230009, Anhui, PR China

[b] Key Laboratory of Process Optimization and Intelligent Decision-Making (Ministry of Education), Hefei University of Technology, Hefei 230009, Anhui, PR China

ARTICLE INFO

Article history:
Received 17 June 2016
Revised 6 September 2016
Accepted 4 October 2016

Keywords:
Individual reputation
Public goods game
Spatial reciprocity
Promotion of cooperation
Social governance

ABSTRACT

How to model the evolution of cooperation within the population is an important and interdisciplinary issue across the academia. In this paper, we propose an improved public goods game model with reputation effect on spatial lattices to investigate the evolution of cooperation regarding the allocation of public resources. In our model, we modify the individual utility or fitness as a product of the present payoff and reputation-related power function, and strategy update adopts a Fermi-like probability function during the game evolution. Meanwhile, for an interaction between a pair of partners, the reputation of a cooperative agent will be accrued beyond two units, but the defective player will decrease his reputation by one unit. Extensive Monte Carlo numerical simulations indicate the introduction of reputation will foster the formation of cooperative clusters, and greatly enhance the level of public cooperation on the spatial lattices. The larger reputation factor leads to the higher cooperation level since the reputation effect will be enormously embedded into the utility evaluation under this scenario. The current results are vastly beneficial to understand the persistence and emergence of cooperation among many natural, social and synthetic systems, and also provide some useful suggestions to devise the feasible social governance measures and modes for the public resources or affairs.

1. Introduction

As is well known that the peace and development is the topic of the modern society and social conflicts over the world have been greatly declined [1]. To date, the collective cooperation has become a widespread phenomenon among the population ranging from cellular organisms, to vertebrates, human beings, and even regions or countries [2]. However, from the perspective of Darwinian evolutionary theory [3], any behavior that does not donate to oneself will not prevail. Thus, how to illustrate this kind of inconformity between the theory and reality is a challenging task met by scientists from inter-disciplinary fields including social, engineering and natural sciences [4]. Among them, the evolutionary game theory [5] provides a powerful framework to model the evolution of collective behaviors within many natural, social, biological, economical and even engineering or man-made systems.

In the standard setup of evolutionary game theory [6,7], players or agents within the population interact according to the specific rules related with the game model, and it is also assumed that each individual needs to play a game with any other one (*i.e.*, well-mixing topology hypothesis). However, the topological structure of real systems often deviates from the well-mixed scenarios. In particular, Nowak and May [8] seminally investigated the behavior of prisoner's dilemma game on a spatial square lattice where each agent can only play with his nearest neighbors, and found that cooperators can organize into compact clusters to avoid the exploitation of defectors under the large defection temptation. Starting from this work, various game mechanisms, such as social diversity [9,10], utility weight [11,12], reward and punishment [13–18], individual mobility [19–22] and so on, are put into the spatial lattices, it is convincingly indicated that the cooperation can be further promoted into a higher level. Recently, large quantities of evidences prove that many real-world systems exhibit the obvious small-world effect [23] and scale-free property [24], that is, the topology exhibited in real systems is neither random nor regular, and the topology heterogeneity is pretty prevalent within these systems [25,26]. Santos et al. [27] studied the evolution of cooperation on scale free topology and found that the cooperation can still emerge evens under the condition in which $T > 2S$, meanwhile the results forcefully demonstrate that the scale-free networks provide a unifying topological foundation for the emergence of coopera-

* Corresponding author at: School of Management, Hefei University of Technology,Hefei 230009, China.
E-mail address: dingshuai@hfut.edu.cn (S. Ding).

http://dx.doi.org/10.1016/j.chaos.2016.10.003

tion, and extensive works are devoted to illustrating the evolutionary dynamics on complex networks [28–34].

As pointed out by Nowak in Ref. [35], network and spatial reciprocity along with kin selection [36], direct [37] and indirect reciprocity [38,39], and group interactions [40], has macroscopically become viable means to promote the collective cooperation between individuals. Microscopically, the specific mechanisms to enhance the cooperation still deserves to be studied in depth [41–43]. As an example, the reputation is very significant for an individual to survive in the society, especially within the financial and academic communities, and the individual reputation usually builds from past behaviours and works. On one hand, it is very difficult for a person to obtain the bank loan if he owns a bad credit in the past and needs to apply for the financial service for the present. On the other hand, once a researcher has a very low credit score within the academic communities, he may be hard to acquire the research grants and even lose his post. In the meantime, the reputation effect is also extremely significant for us to design the feasible measures for the social governance and public management affairs [44,45]. Thus, exploring the role of reputation effect in the evolution of cooperation is an important issue in the field of evolutionary game theory, related works make some attempts. Among them, Nowak and Sigmund [38] firstly proposed the indirect reciprocity based on the image score to promote the cooperation, where the image score can be considered as the prototype of reputation. Milinski et al. [46] proved that the cooperation can be sustained into a high level by alternating the public goods and indirect reciprocity game through a large number of experiments. Starting from these works, Fu et al. [47] designed a partner switching rule on the basis of reputation during the evolutionary process, and plenty of numerical simulations demonstrated that cooperation would prevail in the population. Furthermore, Wang et al. [48] presented a novel reputation inferring scheme to be applied into the prisoner's dilemma game (PDG) and snowdrift game (SDG), where the cost and error of information dissemination regarding the strategy of players is considered, and found that this kind of limited inferring ability could enormously promote the level of cooperation when compared to the traditional PDG [49–51] or SDG [52,53]. Chen et al. [54] proposed an adaptive reputation assortment mechanism to characterize the individual diversity and found that the emergence of cooperation can be induced by the reputation assortment. So far, however, the public goods game (PGG) model integrating the reputation into the utility evaluation is still absent, and here we try to introduce an improved utility evaluation based on the reputation effect in the spatial public goods game to deeply explore the impact of individual reputation on the cooperative behaviors. Large quantities of numerical simulations indicate that the cooperation can be highly promoted in comparison with the standard PGG without any reputation setting, and the current results will be conducive to further understanding the tragedy of commons, and even help the related administration to propose some effective strategies to deal with the public resource and benefits.

The rest of this paper is structured as follows. Section 2 introduces the improved PGG with reputation effect in detail. A large plethora of simulations will be provided in Section 3 so that we can carefully analyze the role of reputation in the promotion of cooperation. At last, the concluding remarks will be summarized in Section 4.

2. The PGG model with individual reputation on spatial lattices

The current model starts from the traditional PGG model and the individual reputation will be integrated into the PGG model. For the initial setup, $N = L \times L$ players are allocated on a spatial square lattice, in which each intersection can only contain one player. Each player (say, x) will be assigned as a cooperator (C, $s_x = 1$) or defector (D, $s_x = 0$) with the equal probability (i.e., 50%). Meanwhile, player x will be endowed with a reputation value R_x, which is a random value uniformly taken from the interval [1, R_{max}]. Without lacking the generality, we set the biggest reputation value $R_{max} = 100$, that is, $R_x \in [1, 100]$.

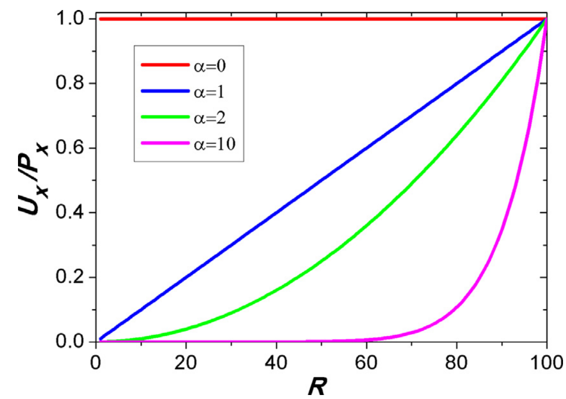

Fig. 1. Relationship between the normalized utility and the reputation value, where the reputation factor is set to be 0, 1, 2, and 10, repectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

After that, each player x will perform the PGG interaction with his k nearest neighbours and accumulate the payoffs through the game round. In our model, every player x needs to participate in $k+1$ PGG groups which include one group centering on himself and other k groups focusing on his k nearest neighbors. Here, we denote G as the set of PGG groups that player x needs to take part in, and g represents one of groups. P_c^g or P_d^g denote the payoff obtained by player x as a cooperator or a defector from PGG group g, respectively, and can be calculated as follows

$$\begin{cases} P_d^g = \dfrac{rn_C}{k+1} = \dfrac{r\sum_{x\in g} s_x}{k+1} \\ P_c^g = P_d - 1 \end{cases} \tag{1}$$

where n_C is the total number of cooperators within PGG group g during this game round, k is the number of nearest neighbors, and $r \geq 1.0$ denotes the synergy factor which will give the population to the incentives to cooperate. Accordingly, the total payoff (P_x) that player x can obtain from $k+1$ PGG groups can be aggregated as follows

$$P_x = \sum_{g\in G} P_x^g \tag{2}$$

where P_x^g represents the payoff of player x within PGG group g. In order to take the individual reputation into account, we consider the individual utility (U_x) as the product of individual total payoff and reputation value-related power function, and U_x can be written as

$$U_x = P_x \left(\frac{R_x}{R_{max}} \right)^{\alpha} \tag{3}$$

where α is a tunable parameter which denotes the extent of reputation embedded into the utility expression, and in what follows, α is often called the reputation factor. Additionally, in this paper, the reputation factor is set to be $\alpha \geq 0$ so as to encourage the agents to take the cooperation strategy (Theoretically, α can be also set to be negative but it is meaningless for the promotion of cooperation). Henceforth, the total payoff P_x does not completely characterize the individual utility, P_x and reputation value R_x will be combined to compute U_x in the present model. Furthermore, we illustrate the relationship between the normalized utility (U_x/P_x) and R in Fig. 1, which implies that the large the reputation factor α, the more slowly the normalized utility (that is, the value of