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Complex dynamics and multistability with increasing rationality in market games

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ABSTRACT

In this work we study oligopoly models in which firms adopt decision mechanisms based on best response techniques with different rationality degrees. Firms are also assumed to face resource or financial constraints in adjusting their production levels, so that, from time to time, they can only increase or decrease their strategy by a bounded quantity. We consider different families of oligopolies of generic sizes, characterized by heterogeneous compositions with respect to the rationality degrees of firms. We analytically study the local stability of the equilibrium depending on the oligopoly size and composition and through numerical simulations we investigate the possible dynamics arising when trajectories do not converge toward the equilibrium. We show that in this case complex dynamics can arise, and this is due to both the loss of stability of the equilibrium and to the emergence of multiple attractors, with the stable steady state coexisting with a different, periodic or chaotic, attractor. In particular, we show that multistability phenomena occur when the overall degree of rationality of the oligopoly is increased. Finally, we investigate the effect of non-convergent dynamics on the realized profits.

1. Introduction

Due to the complexity of an oligopolistic market setting, firms should be more realistically assimilated to reduced rationality players, which only have a partial knowledge of the market and of their competitors' strategies. In classical market games, firms can compete with respect to the price of the good (Bertrand oligopoly [1]) or to the quantity to produce (Cournot oligopoly [2]). In latter case, under the assumption of bounded rationality, firms can only try to dynamically adapt their production levels toward the profit maximizing strategy, in a sort of evolutionary approximation of the Nash equilibrium. In the last twenty five years a very wide research strand has focused on the study of the dynamical properties of models describing market games among boundedly rational firms (for an in-depth review about possible decision mechanisms and oligopoly modeling, we refer to the book of Bischi et al. [3]). In particular, the dynamical adjustment of production levels may not converge toward the Nash equilibrium [2], following periodic, quasi-periodic and chaotic dynamics. Among the reasons for the occurrence of complex dynamics, we can mention the boundedly rational nature of the firms, the presence of nonlinearities in the market functions, behavioral or technological heterogeneities among firms and the oligopoly size. Restricting to the most recent works, we can mention the contributions about heterogeneous duopolies [4–8], oligopolies of fixed, small sizes [9–12] and oligopolies of generic sizes, with both fixed [13] and variable compositions [13–17].

In the present work, we consider families of oligopolies of generic size N obtained from heterogeneous combinations of three kinds of players, who have in common the decision mechanism, which is based on best response techniques, but who differ in their respective degrees of rationality. Firstly, for each model, we study the local stability of the steady state, which coincides with the Nash equilibrium. We analytically show for which oligopoly compositions and sizes the equilibrium is locally asymptotically stable. In particular, we prove that, for a suitably large fraction of rational players, we can have heterogeneous oligopolies whose equilibrium remains stable as their size increases. We then investigate through simulations the occurrence of complex dynamics. We show that when the equilibrium loses its stability, periodic and chaotic output trajectories can arise. More significantly, we show that even if the equilibrium is locally stable, we can have trajectories that converge toward a different attractor. Usually, multistability in the oligopolistic literature is connected to the technological heterogeneity of firms [5]. To the best of our knowledge, this is the first example of coexistence between differently complex

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attractors in oligopolies of generic sizes in which the only heterogeneity among firms concerns the rationality degree. In particular, we show that such level of complexity occurs increasing the overall rationality degree of the oligopoly, namely the fraction of rational players. This means that rational players can actually have a destabilizing effect, in the sense that the globally stable equilibrium can become just locally stable increasing their number.

Finally, we investigate the average profits achieved by each different kind of firms, in particular when trajectories do not converge toward the equilibrium. We show that, in such cases, realized profits are larger than equilibrium profits, so that instability can end in an advantage for firms. Moreover, especially when nonconverging dynamics are due to multistability phenomena, it is no longer true that more rationality means larger profits.

The remainder of the paper is organized as follows. In Section 2, after presenting the economic setting, we introduce and describe LMA, Nash and rational players. In Section 3, we present four families of heterogeneous oligopolies and we analytically study the local stability of the equilibrium. In Section 4, we present numerical investigations. Conclusions and possible future researches are reported in Section 5.

2. Cournot market games

Let us consider an industry consisting of N firms, indexed by $i = 1, \dots, N$, which produce quantities q^i of homogeneous goods and have linear cost functions $C(q^i) = cq^i$, where $c > 0$ represents the identical, constant marginal cost of each oligopolist. We assume that the price of the goods depends on the total output Q through the isoelastic inverse demand function $p(Q) = 1/Q$ (more details can be found in [2]).

The profit of the ith firm is then

$$\pi_i(q^i, Q^{-i}) = \frac{q^i}{q^i + Q^{-i}} - cq^i, \tag{1}$$

where Q^{-i} indicates the aggregate output level of all the oligopolists but the ith one. Without loss of generality, from now on we can assume $c = 1$. We notice that this means that all the analytical results presented in this section and the qualitative behavior of the numerical simulations reported in Section 4 are independent of c, which just rescales trajectories (setting $c = 1$ actually corresponds to the change of variable $q^i := cq^i$.) The previous framework sets up a game, in which players are the N oligopolists, the set of admissible strategies consists of positive production levels $q^i > 0$ and payoff functions are profit functions (1). Ahmed and Agiza [18] and Matsumoto and Szidarovski [19] showed that such game has only one Nash equilibrium $(q^*, \dots, q^*) \in \mathbb{R}^N$ with

$$q^* = \frac{N-1}{N^2}, \tag{2}$$

while the equilibrium aggregated quantity Q^*, profit π^* and price p^* are

$$Q^* = \frac{N-1}{N}, \quad \pi^* = \frac{1}{N^2}, \quad p^* = \frac{N}{N-1}. \tag{3}$$

In the oligopolies we aim to study, all the agents, who make their production choices at each discrete time $t \in \mathbb{N}$, adopt decision mechanisms based on best response techniques and differ in their rationality degrees, in particular with respect to the informational endowment. This means that some kinds of players have to adapt their production decisions at each time step, accordingly to an adjustment mechanism which depends on their, possibly reduced, informational endowment and on the, possibly variable, output levels of their competitors. This means that the difference $|\tilde{q}_{t+1} - q_t|$ between the next period output level $\tilde{q}_{t+1}$ given by the adjustment rule they adopt and the current production decision q_t may be very large. However, in real situations, firms can meet constraints in trying to adapt their production levels. For example, they may be not able to immediately modify their output decision of any quantity. This is in general due to capacity and financial constraints or shortage of manpower, which prevent increasing the production level arbitrarily, as well as for economies of scale and break-even considerations or the impossibility to dismiss labor force, which do not allow firms to excessively decrease output decisions. A discussion about firms' behavior can be found in the book by Sterman [20]. For the reasons given above, it is more suitable to assume that firms gradually adapt their strategies toward the production level $\tilde{q}_{t+1}$. This can be modeled through the following adjustment mechanism

$$q_{t+1} = q_t + \sigma(\tilde{q}_{t+1} - q_t), \tag{4}$$

where $\sigma : \mathbb{R} \to \mathbb{R}$ is a differentiable, strictly increasing and bounded function which describes the feasible production variation. We assume $\sigma(0) = 0$, so that steady states of the original adjustment mechanism are preserved and $\sigma'(0) = 1$. As we will see, this last assumption ensures that the local stability of the steady state is not affected by the particular choice of σ. Finally, we impose that $\sigma(x)$ is convex for $x < 0$ and concave for $x > 0$. This, together with the previous requirements, means that the more $\tilde{q}_{t+1}$ is different from q_t, the more q_{t+1} is different from $\tilde{q}_{t+1}$. This allows mimicking that the output level adjustment is increasingly more difficult to realize as $|\tilde{q}_{t+1} - q_t|$ becomes large. We notice that similar gradual adjustment mechanisms have been proposed and used in oligopoly modeling [13,15,21]. Finally, we stress that it is possible to show that if no output limiter was considered, the players we are going to consider would choose to produce constantly null production levels for any sufficiently large oligopoly size (actually, approximatively $N > 9$), which would mean that all firms would leave the market, which is indeed unrealistic.

To complete the description of the market game, we need to precise the assumptions on each kind of player. Firstly, we assume that players characterized by the same informational endowment are identical. In particular, this means that the starting production choice of such players is the same, and, consequently, production choices of such players coincide at each time. Moreover, each firm is supposed to know the oligopoly composition and the oligopoly size N, which are both constant in time. In what follows we consider three different kinds of players: local monopolistic approximation (LMA), Nash and rational players. LMA players adopts the so-called local monopolistic approximation, which is a boundedly rational mechanism, proposed and studied by Bischi et al. [22]. LMA firms are assumed to have a reduced informational endowment: they only know the current market price, their own current production quantity and they have a local knowledge of the demand function at the current price, obtained, for example, through market experiments. Nash players have complete information about the price function and the cost functions and make the assumption that all the other players have perfect foresight and use a best response mechanism. Moreover, they have enough computational capabilities to compute the Nash equilibrium (2). Finally, rational players know the demand function, the cost functions of each player and are able to compute the optimal output level which maximizes their profits with respect to the expected strategies of the other players. In particular, we assume that rational players are able to exactly foresight the production decisions that LMA and Nash players will adopt for the next period.

The remainder of this Section is devoted to the description of the discrete equations governing the adjustment mechanisms of each players. We notice that each firm is assumed to know the oligopoly composition and the oligopoly size N, which are both constant in time.