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Nonlinear analysis of a parametrically excited beam with intermediate support by using Multi-dimensional incremental harmonic balance method

Shihua Zhou, Guiqiu Song*, Zhaohui Ren, Bangchun Wen

School of Mechanical Engineering & Automation, Northeastern University, 110819, China

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ABSTRACT

In this paper, a nonlinear Euler-Bernoulli beam under a concentrated harmonic excitation with intermediate nonlinear support is investigated. Continuous expression for the kinetic energy, potential energy and dissipation function are constructed. An energy method based on the Lagrange equation combined with the Galerkin truncation is used for discretizing the governing equation. The Multi-dimensional incremental harmonic balance method (MIHBM) is derived, and the comparisons between the numerical results and the approximate analytical solutions based on the MIHBM verify the excellent accuracy of the MIHBM. The steady state dynamic of the beam is investigated by MIHBM. In order to investigate the energy transmission and understand the vibration response of the Euler-Bernoulli beam, the effects of the key parameters on the dynamic behaviors are studied and discussed, individually. The results show that the amplitude-frequency curves exhibits softening nonlinear behavior in the super-harmonic resonance region, and near resonant region the hardening nonlinear behavior is observed depending on the different parameters. Nonlinear dynamic analysis, such as bifurcation, 3-D frequency spectrum, waveform, frequency spectrum, phase diagram and Poincaré map, are also presented in order to study the influences of the key parameters on the vibration behaviors for the beam in a more accurate manner. In addition, the path to chaotic motion is observed to be through a sequence of the periodic motion and quasi-periodic motion.

1. Introduction

Due to the development of engineering technology, the bridge and vibration platform, modeled as beam, are widely used in many technological devices and machines components so that scientists and engineers have the strong interest to study the unwanted vibration and resonance. The vibration characteristics and stability of the beam are highly affected by the external excitation and support features. Therefore, understanding the dynamic responses of a beam resting on a nonlinear foundation subjected to a concentrated load has attracted much attention over many years. The dynamic characteristics are affected by a number of parameters individually or their combinations such as the influences of different boundary conditions, parametric instabilities of beam caused by time-varying load, nonlinear support effect on stability, different analytical and numerical solution techniques and various other parameters affecting the dynamic behaviors of the beam system. The motivation of this paper is to understand the dynamic responses of the beam interacting with nonlinear foundation.

The dynamic behaviors of beam have been extensively investigated in the past decades by researchers with experimental and theoretical analysis. Han [1] presented four models for the transversely vibrating uniform beam with the Euler-Bernoulli, Rayleigh, shear and Timoshenko. And a numerical example was shown for a non-slender beam to signify the differences among the four beam models. Pellicano [2] analyzed the dynamic behaviors of a simply support beam subjected to an axial transport of mass. Linear subcritical behavior, bifurcation analysis and stability and direct simulation of global postcritical dynamics were presented. Öz et al. [3] applied the multiple scales method to research the nonlinear vibration and stability of axially moving beam and tensioned pipes conveying fluid with time-dependent velocity. Riedel [4] studied the forced vibration of a nonlinear axially moving strip with coupled transverse and longitudinal motions by applying the method of multiple scales to assess the effects of speed and the internal resonance. Yang [5] investigated the bifurcation and chaos of an axially accelerating viscoelastic beam by adopting the Kelvin-Voigt model. The effects of the mean axial speed, the amplitude of speed

* Corresponding author.
E-mail addresses: zhou_shihua@126.com (S. Zhou), guiqiusong@126.com (G. Song), zhhren.neu@gmail.com (Z. Ren), bangchunwen@126.com (B. Wen).

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fluctuation and the dynamic viscosity coefficient were discussed. Awrejcewicz [6,7] derived the mathematical modeling and analyzed the spatio-temporal chaotic behaviors of flexible simple and curved Euler-Bernoulli beam. The phenomena of transition from symmetric to asymmetric vibrations were studied and explained. Han [8] discussed the dynamic behaviors of a nonlinear elastic beam with large deflection. The chaotic characteristic was investigated and the comparison between single and double mode models was carried out. Amer [9] used the method of multiple scales to investigate the nonlinear behavior of a string-beam coupled system subjected to parametric excitation, and the multiple solutions propertied and jump discontinuous phenomenon were presented. Kahrobaiyan [10] presented a size-dependent functionally graded Euler-Bernoulli beam model based on the strain gradient theory. The static and free-vibration of the model were investigated in which the properties were varying through the thickness according to a power law and the results were compared. Ghayesh [11] studied the nonlinear dynamics of a forced axially moving viscoelastic beam by using the pseudo-arc length continuation technique and a direct time integration, and the amplitude-frequency responses and bifurcation diagrams were presented for several values of the system parameter. Hamed [12] presented the nonlinear behaviors of a string-beam coupled system subjected to external, parametric and tuned excitations. The effects of the different parameters on both responses and stability of the system were investigated. Ghayesh [13–15] developed the forced nonlinear dynamics of an axially moving beam with coupled longitudinal and transverse displacements. The effects of system parameters on the frequency-response curves and the bifurcation diagrams were presented.

Many contributions on the vibration and stability of the beam system resting on a nonlinear foundation were presented. Kargarnovin [16] studied the dynamic response of a Timoshenko beam with infinite length supported by a generalized Pasternak-type viscoelastic foundation. A parametric study was carried out for an elliptical load distribution and the effects of the load speed and frequency on the beam responses were displayed. Chen [17] investigated the bifurcation and chaotic motion of higher-dimensional nonlinear system for the nonplanar nonlinear vibrations of an axially accelerating moving viscoelastic beam. The influences of the mean axial velocity, the amplitude of velocity fluctuation and the frequency of velocity fluctuation were discussed. Ansari [18] studied the vibration of an Euler-Bernoulli beam, resting on a nonlinear Kelvin-Voight viscoelastic foundation, traversed by a moving load in the frequency domain. A conventional railway track was dynamically simulated and the jump phenomenon was observed for three harmonics. Ding [19,20] analyzed the natural frequencies of planar vibration of axially moving beam in the supercritical ranges. The results indicated that the nonlinear coefficient had little effect on the natural frequency. Koroma [21] developed a method to analyze a beam that was continuously supported on a linear nonhomogeneous elastic foundation, and exhibited multiple resonances corresponding to the foundation stiffness of individual sections. Yan [22,23] researched the dynamic behaviors of an axially accelerating viscoelastic Timoshenko beam with the external harmonic excitation, and the bifurcation diagrams were presented to discuss the effect of the external transverse excitation. Jorge [24] presented the dynamic response of beam resting on nonlinear elastic foundation, subjected to moving loads. The effects of the load's intensity, velocity and the foundation's stiffness were investigated. Sahoo [25,26] analyzed the nonlinear transverse vibration of an axially moving beam subjected to two frequency excitation. The amplitude-frequency curves, stability and bifurcation were obtained using continuation algorithm. Norouzi [27] discussed the chaotic motion of beam resting on a foundation with nonlinear stiffness. And then a parametric study was carried out and the effects of linear and nonlinear parameters on the chaotic behaviors were studied. Vladimir [28] investigated geometrically nonlinear vibrations of a Timoshenko beam resting on a nonlinear Winkler and Pasternak elastic foundation with variable discontinuity. The comparison of the results with various stiffnesses of nonlinear elastic supports of the Winkler and Pasternak type was presented.

The beam system are constantly subjected to different adornments, such as intermediate support, energy sink, vehicle and other machine components at different locations along the lengths, which have an important effect on the dynamics. Therefore, the effects of the adornments on the vibrations and stability of beam system have been investigated in the literature. Pakdemirli [29] investigated the nonlinear vibration of a simply supported stationary beam with an attached cubic nonlinear spring-mass system taking into account the effects of beam midplane stretching and damping. Öz [30] studied the free and forced vibrations of the slightly curved beam resting on a nonlinear elastic foundation. The effects of nonlinear elastic foundation and curvature on the vibrations for the beam were examined. Chakraborty [31] discussed the vibration responses of a nonlinear travelling beam with an intermediate guide. The nonlinear term was very sensitive to the location of the guide if the guide stiffness was small. Darabi [32] analyzed the free vibration of a beam-mass-spring system with the effects of the spring and the point mass as internal boundary conditions, and analytical and numerical results were discussed. Pakdemirli [33] established a simply supported Euler-Bernoulli beam with an intermediate support. And the non-ideal boundary conditions were applied to the beam problem. Ghayesh [34] investigated the forced nonlinear vibrations of an axially moving beam fitted with an intra-span spring support. The sub-critical response was examined when the excitation frequency was set near the first natural frequency for the system with and without internal resonances. Kazemirad [35] examined the nonlinear vibration and stability of a hinged-hinged axially moving beam, additionally supported by a nonlinear spring-mass subjected to a transverse harmonic excitation force as well as a thermal loading. Kesimli [36] established the multi-supported axially moving beam, the effects of axial velocity, constant of spring and the position of the support on the bifurcation points were investigated. Dai [37] developed a new nonlinear theoretical model for cantilevered microbeams subjected to harmonic excitation with an intermediate linear spring support and explored the nonlinear dynamics based on the modified coupled stress theory. In most of the literature concerning the nonlinear vibration of beam, it is rare to found that the system contains both quadratic and cubic nonlinearities at the same time.

The incremental harmonic balance method (IHBM) is an efficient and reliable method for analyzing strongly nonlinear vibration system. It was developed and successfully applied to the analysis of periodic nonlinear structural vibration and related problem by Lau [38]. Huang [39,40] presented the nonlinear vibration analysis of a curved beam subjected to uniform base harmonic excitation with both quadratic and cubic nonlinearities, and investigated the effects softening stiffness, hardening stiffness and modal energy transfer. Sze [41] used the IHBM to formulate the nonlinear vibration analysis of axially moving beam with internal resonance. Cheung [42] applied the IHBM to study the vibration system with cubic nonlinearity, which governed a wide range of engineering problems such as large-amplitude vibration of beams or plates. Cai [43] studied the flutter of system with multiple structural strong nonlinearities by IHBM, and then the bifurcation, limit cycle flutter phenomena and the number of harmonic terms were investigated. Pirmoradian [44] investigated the dynamics of a Timoshenko beam excited by a sequence of identical moving masses using the IHBM. The influences of employing different deformation theories on the critical parameter values of stability and resonance curves were studied. The MIHBM is a very effective and reliable method for solving nonlinear system, but little work