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Stabilization of time-delay neural networks via delayed pinning impulses☆

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ABSTRACT

This paper studies the pinning stabilization problem of time-delay neural networks. A new pinning delayed-impulsive controller is proposed to stabilize the neural networks with delays. First, we consider the general nonlinear time-delay systems with delayed impulses, and establish several global exponential stability criteria by employing the method of Lyapunov functionals. Our results are then applied to obtain sufficient conditions under which the proposed pinning controller can exponentially stabilize the time-delay neural networks. It is shown that the global exponential stabilization of delayed neural networks can be effectively realized by controlling a small portion of neurons in the networks via delayed impulses, and, for fixed impulsive control gain, increasing the impulse delay or decreasing the number of neurons to be pinned at the impulsive moments will lead to high frequency of impulses added the corresponding neurons. Numerical examples are provided to illustrate the theoretical results, which demonstrate that our results are less conservative than the results reported in the existing literatures when the proposed pinning controller reduces to the delayed impulsive controller.

1. Introduction

Neural networks (NNs) are a family of statistical learning models inspired by the central nervous systems of animals (see, [1]). NNs are generally presented as systems of densely interconnected simple elements which model the biological neurons, and send (or receive) messages to (or from) each other. In recent decades, the research on NNs has attracted numerous researchers attentions. This mainly due to their broad applications in many areas including image processing and pattern recognition (see, e.g., [2,3]), data fusion [4], odor classification [5], and solving partial differential equations [6].

In real-world applications, it is inevitable for the existence of time delay in the processing and transmission of signals among neurons of NNs. Hence, it is practical to investigate NNs with time-delay (DNNs) (see, e.g., [7–12]). Stability of DNNs, as a prerequisite for their applications, has been studied extensively in the past decades, and various control methods have been introduced to stabilize the DNNs, such as intermittent control [9], sliding mode control [10], impulsive control [11], and sampled-data control [12]. Among these control algorithms, the impulsive control method has been proved to be an effective approach to stabilize the DNNs. The control mechanism of this method is to control the neuron states of a NN with small impulses which are small samples of the state variables of the NN at a sequence of discrete moments. On the other hand, the time delay is unavoidable in sampling and transmission of the impulsive information in dynamical systems. Therefore, many control problems of dynamical systems have been investigated via delayed impulses in recent years, such as stabilization of stochastic functional systems [13,14], synchronization of dynamical networks [15], and stability analysis of nonlinear impulsive and switched time-delay systems with delayed impulses [16].

The regular impulsive control method to stabilize a NN is to control each neuron of the network to tame the neuron dynamics to approach a steady state (i.e., equilibrium point). However, a NN is normally composed of a large number of neurons, and sometimes it is expensive and infeasible to control all of them. Motivated by this practical consideration, the idea of controlling a small portion of neurons, named pinning control, was introduced in [17,18], and many pinning impulsive control algorithms have been reported for many control problems of dynamical networks (see, e.g., [19–24]) It is worth noting that no time delay is considered in these pinning impulsive controllers proposed in the above literatures. However, it is natural and essential to consider the de-

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lay effects when processing the impulse information in the controller.

Due to the cost effectiveness advantage of impulsive control method and pinning control strategy and the wide existence of time delay, it is practical to investigate the pinning impulsive control approach that takes into account of impulse delays. However, to our best knowledge, no corresponding result has been reported for stabilization of DNNs. Therefore, in this paper, we propose a novel pinning delayed-impulsive controller for the DNNs. First, we use Lyapunov functional method to construct new criteria for global exponential stability of general nonlinear differential functional equations with delayed impulses. Our results are then used to establish sufficient conditions to guarantee the proposed controller can exponentially stabilize the DNNs.

Compared with the existing work in the literature, the main contributions of our work are summarized as follows.

- By the method of Lyapunov–Krasovskii functionals, several novel global exponential stability criteria are constructed for the general nonlinear time-delay systems subject to delayed impulses. It is shown that the delayed impulses can be applied to stabilize the unstable time-delay systems. Therefore, it is guaranteed that the time-delay NNs can be stabilized by well-designed delayed impulsive controllers according to our results. Recently, various stability results of the time-delay systems with delay impulsive effects have been reported in the literature by using the Lyapunov–Razumikhin technique (see, e.g., [13,14]). However, it is worth noting that, as pointed out in [25] (Chapter 4.8, page 254), the Lyapunov–Razumikhin method can be considered as a particular case of the Lyapunov–Krasovskii method, and the latter approach sometimes is more general than the former method. Hence, it is worthwhile to conduct stability analysis of time-delay systems with delayed impulses using the method of Lyapunov–Krasovskii functionals.
- We propose a new pinning delayed-impulsive controller in the following form

$$U_i(t,x_i) = \begin{cases} \sum_{k=1}^{\infty} q x_i(t-d)\delta(t-t_k^-), & i \in \mathfrak{D}_k^l, \\ 0, & i \notin \mathfrak{D}_k^l, \end{cases} \tag{1}$$

where U_i is the impulsive input to the ith neuron, l denotes the number of neurons to be controlled at each impulsive instant, and $\mathfrak{D}_k^l$ is a index set which is associated with the pinning algorithm and will be introduced in Section 2 in detail. At impulsive time $t=t_k$, it can be seen that only l neurons are controlled. The set $\mathfrak{D}_k^l$ is related to the pinning algorithm which stems from [19], and has been successfully applied to the control problem of various dynamical networks (see, e.g., [26–30]). However, no time-delay has been considered in these results. Therefore, the pinning algorithm introduced in [19] can be treated as a particular case of our pinning delayed-impulsive control strategy (i.e., $d=0$). It is worth noting that the existence of time delay in controller (1) brings dramatic difficulties to estimate the relation between the states $x_i(t_k^-)$ and $x_i(t_k-d)$, and then guarantee the delayed impulses contribute to the stabilization process of DNNs. Though, recent studies in [31] and [32] have considered the delay state $x_i(t_k-d)$ in the pinning impulsive controller, the controller depends on both the state $x_i(t_k)$ and $x_i(t_k-d)$, and there is no theoretical analysis of how the delay state $x_i(t_k-d)$ affects the pinning control process. Actually, results in [32] have shown that the delay states can either contribute to the stability of the system or act as disturbances to the dynamical system. To our best knowledge, this is the first time that a pinning impulsive controller is proposed with delayed impulse effects which depend only on the delay state $x_i(t_k-d)$. The detailed discussion of the delay effects on the stabilization process of DNNs can be found in Section 4.
- If l is equal to n (the number of neurons in the NN), the pinning controller (1) reduces to the linear delayed impulsive controller in the form

$$U_i(t,x_i) = \sum_{k=1}^{\infty} q x_i(t-d)\delta(t-t_k^-), \quad \text{for } i=1,2,\ldots,n, \tag{2}$$

which implies that all the neurons are controlled at every impulse moment. Although delayed impulsive controller (2) has been studied in [33] and [34], our stabilization results are less conservative in the sense that we can obtain a larger upper bound for impulse delay d for given impulsive control gain q and fixed impulsive interval length (i.e., t_k-t_{k-1} is constant for all $k\in\mathbb{N}$).

The remainder of this paper is organized as follows. In Section 2, we formulate the problem and introduce the pinning delayed-impulsive control algorithm. In Section 3, global exponential stability results of general nonlinear impulsive systems with time-delay are obtained, and a numerical example of linear impulsive equation with delay is considered to illustrate these results. Then, in Section 4, stability criteria obtained in Section 3 are applied to construct sufficient conditions under which the proposed controller can exponentially stabilize the DNNs. The efficiency of the proposed results is demonstrated by numerical simulations in Section 5. Finally, conclusions are stated in Section 6.

2. Preliminaries

Let $\mathbb{N}$ denote the set of positive integers, $\mathbb{R}$ the set of real numbers, $\mathbb{R}^+$ the set of nonnegative real numbers, and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm. For $\alpha,\beta\in\mathbb{R}$, the floor function $\lfloor\alpha\rfloor$ gives the largest integer less than or equal to α, and define $mod(\alpha,\beta) := \alpha - \lfloor\frac{\alpha}{\beta}\rfloor\beta$. For $a,b\in\mathbb{R}$ with $a<b$ and $S\subseteq\mathbb{R}^n$, we define

$$\begin{aligned}\mathcal{PC}([a,b],S) = \Big\{\psi : [a,b]\to S \Big| \psi(t)=\psi(t^+),\ &\text{for any } t\in[a,b);\ \psi(t^-) \\ &\text{exists in } S,\ \text{for any } t\in(a,b];\ \psi(t^-)=\psi(t) \text{ for all but} \\ &\text{at most a finite number of points } t\in(a,b]\Big\},\end{aligned}$$

$$\mathcal{PC}([a,\infty),S) = \Big\{\psi : [a,\infty)\to S \Big| \text{ for any } c>a,\ \psi|_{[a,c]}\in\mathcal{PC}([a,c],S)\Big\},$$

where $\psi(t^+)$ and $\psi(t^-)$ denote the right and left limit of function ψ at t, respectively. For a given constant $\tau>0$, the linear space $\mathcal{PC}([-\tau,0],\mathbb{R}^n)$ is equipped with the norm defined by $||\psi||_\tau = \sup_{s\in[-\tau,0]}||\psi(s)||$, for $\psi\in\mathcal{PC}([-\tau,0],\mathbb{R}^n)$.

Consider the following DNN:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-r)) + J_i, \tag{3}$$

for $i=1,2,\ldots,n$, where $x_i\in\mathbb{R}$ is the state of the ith neuron; n denotes the number of neurons in DNN (3); $f_j(x_j(t))$ denotes the output of the jth neuron at time t; constants a_{ij} and b_{ij} represent the strengths of connectivity between neurons i and j at time t and $t-r$, respectively; r corresponds to the transmission delay when processing information from the jth neuron; constant J_i denotes the external bias or input from the outside of the network to the ith neuron; constant c_i denotes the rate with which the ith neuron will reset its potential when disconnected with the other neurons of the network and external input.

Throughout this paper, we make the following assumptions:

(A_1) $f_i(0)=0$ and there exists a constant L_i such that $|f_i(u)-f_i(v)|\le L_i|u-v|$ for all $u,v\in\mathbb{R}$ and $i=1,2,\ldots,n$;

(A_2) $J_i=0$ for $i=1,2,\ldots,n$.