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# Wave Packet Systems on Local Fields

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**Abstract:** In this aper, we introduce the notion of wave packet systems on local fields of positive characteristic and derive some characterizations of these systems by means of two basic equations in the Fourier domain. More precisely, we establish a complete characterization of orthogonal wave packet systems in $L^2(K)$ which include the corresponding results of wavelet analysis and Gabor theory as the special cases. We shall also provide a sufficient condition of the completeness of wave packet systems on local fields of positive characteristic subject to some mild conditions. The paper concludes with the necessary and sufficient conditions for the wave packet systems to be wave packet Parseval frames for $L^2(K)$.

**Keywords:** Wavelet frame; Gabor frame, Parseval frame; wave packet; orthogonal wave packet frame; local field; Fourier transform.

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## 1. Introduction

Signals are in general non-stationary. A complete representation of non-stationary signals requires frequency analysis that is local in time, resulting in the time-frequency analysis of signals. Although time-frequency analysis of signals had its origin almost sixty years ago, there has been major development of the time-frequency distributions approach in the last three decades. The basic idea of the method is to develop a joint function of time and frequency, known as a time-frequency distribution, that can describe the energy density of a signal simultaneously in both time and frequency. Basically, there are two kinds of time-frequency representations. One is the quadratic method covering the time-frequency distributions, and the other is the linear approach including the Gabor systems, wavelet systems and wave packet systems.

Gabor systems were first introduced by Denis Gabor [8] by using a Gaussian distribution function as a window function with the aim of constructing efficient, time-frequency localized expansions of finite-energy signals. Gabor systems $\{E_{mb}T_{na}\psi(x) : m, n \in \mathbb{Z}\}$ are generated by modulations and translations of a single function $\psi(x) \in L^2(\mathbb{R})$ and hence, can be viewed as the set of time-frequency shifts of $\psi(x)$ along the lattice $a\mathbb{Z} \times b\mathbb{Z}$ in $\mathbb{R}^2$. On the basis of this development, Duffin and Schaeffer [7] introduced frames for Hilbert spaces, while addressing some deep problems in non-harmonic Fourier series. Frames did not generate much interest outside non-harmonic Fourier series until the seminal work by