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DEGENERATE, STRONG AND STABLE YANG-MILLS-HIGGS PAIRS

ZHI HU & PENGFEI HUANG

ABSTRACT. In this paper, we introduce some notions on the Hitchin pair consisting of a Chern connection and a Higgs field closely related to the first and second variation of Yang-Mills-Higgs functional, such as degenerate Hitchin pair, (strong) Yang-Mills-Higgs pair, stable Yang-Mills-Higgs pair. We investigate some properties of such pairs under the various contexts.

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1. INTRODUCTION

Since 1950s, Yang-Mills theory first explored by several physicists had a profound impact on the developments of differential and algebraic geometry. A remarkable fruit owed to Donaldson is constructing invariants of 4-manifolds via studying the homology of the moduli space of anti-self-dual $SU(2)$ -connections, where technical challenges come from Uhlenbeck compactification of moduli space and handling singularities through the metric perturbations[1, 2]. In 1987 Hitchin considered the 2-dimensional reduction of the self-dual Yang-Mills equations on \mathbb{R}^4 as a manner of symmetry breaking, then he introduced a (1,0)-form ϕ (valued in complex adjoint vector bundle), called the Higgs field for the Riemann surface, which is described by the so-called Hitchin self-duality equations[3]:

$$\begin{aligned} F_A + [\phi, \bar{\phi}] &= 0, \\ d_A'' \phi &= 0. \end{aligned}$$

Influenced by Hitchin's work, Simpson generalized the conception of Higgs field to the higher dimensional case[4], and he made great innovations in various areas of algebraic geometry[5, 6, 7]. Since then Higgs bundles have emerged in the last two decades as a central object of study in geometry, with several links to physics and number theory.

Let us first recall some basic definitions.

Definition 1.1. ([8, 9, 10]) Let X be an n -dimensional compact Kähler manifold with Kähler form ω , and let Ω_X^1 be the sheaf of holomorphic 1-forms on X . A Higgs sheaf over X is a coherent sheaf E of dimension n over X , together with a morphism $\phi : E \rightarrow E \otimes \Omega_X^1$ of \mathcal{O}_X -modules (that is usually called the Higgs field), such that the morphism $\phi \wedge \phi : E \rightarrow E \otimes \Omega_X^2$ vanishes. A Higgs bundle is a locally-free Higgs sheaf. A subsheaf F of E is called the Higgs subsheaf if $\phi(F) \subset F \otimes \Omega_X^1$, i.e. the pair $F = (F, \phi|_F)$ becomes itself a Higgs sheaf. Let (E_1, ϕ_1) and (E_2, ϕ_2) be two Higgs sheaves over X . A morphism between them is a map $E_1 \rightarrow E_2$ such that the following diagram commutes

$$\begin{array}{ccc} E_1 & \xrightarrow{\phi_1} & E_1 \otimes \Omega_X^1 \\ f \downarrow & & f \otimes 1 \downarrow \\ E_2 & \xrightarrow{\phi_2} & E_2 \otimes \Omega_X^1. \end{array}$$

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