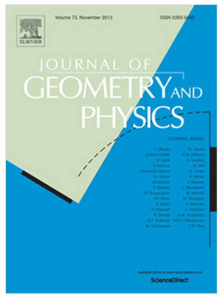
Accepted Manuscript



Degenerate, strong and stable Yang-Mills-Higgs pairs

Zhi Hu, Pengfei Huang

PII: S0393-0440(17)30147-X
DOI: http://dx.doi.org/10.1016/j.geomphys.2017.05.019
Reference: GEOPHY 3010

To appear in: *Journal of Geometry and Physics*

Received date : 7 December 2016
Revised date : 5 May 2017
Accepted date : 29 May 2017

Please cite this article as: Z. Hu, P. Huang, Degenerate, strong and stable Yang-Mills-Higgs pairs, *Journal of Geometry and Physics* (2017), http://dx.doi.org/10.1016/j.geomphys.2017.05.019

DEGENERATE, STRONG AND STABLE YANG-MILLS-HIGGS PAIRS

ZHI HU & PENGFEI HUANG

ABSTRACT. In this paper, we introduce some notions on the Hitchin pair consisting of a Chern connection and a Higgs field closely related to the first and second variation of Yang-Mills-Higgs functional, such as degenerate Hitchin pair, (strong) Yang-Mills-Higgs pair, stable Yang-Mills-Higgs pair. We investigate some properties of such pairs under the various contexts.

CONTENTS

1. INTRODUCTION

Since 1950s, Yang-Mills theory first explored by several physicists had a profound impact on the developments of differential and algebraic geometry. A remarkable fruit owed to Donaldson is constructing invariants of 4-manifolds via studying the homology of the moduli space of anti-self-dual $SU(2)$-connections, where technical challenges come from Uhlenbeck compactification of moduli space and handling singularities through the metric perturbations[1, 2]. In 1987 Hitchin considered the 2-dimensional reduction of the self-dual Yang-Mills equations on $\mathbb{R}^4$ as a manner of symmetry breaking, then he introduced a (1,0)-form ϕ (valued in complex adjoint vector bundle), called the Higgs field for the Riemann surface, which is described by the so-called Hitchin self-duality equations[3]:

$$F_A + [\phi, \overline{\phi}] = 0,$$
$$d''_A \phi = 0.$$

Influenced by Hitchin's work, Simpson generalized the conception of Higgs field to the higher dimensional case[4], and he made great innovations in various areas of algebraic geometry[5, 6, 7]. Since then Higgs bundles have emerged in the last two decades as a central object of study in geometry, with several links to physics and number theory.

Let us first recall some basic definitions.

Definition 1.1. ([8, 9, 10]) Let X be an n-dimensional compact Kähler manifold with Kähler form ω, and let Ω_X^1 be the the sheaf of holomorphic 1-forms on X. A Higgs sheaf over X is a coherent sheaf E of dimension n over X, together with a morphism $\phi : E \to E \otimes \Omega_X^1$ of $\mathcal{O}_X$-modules (that is usually called the Higgs field), such that the morphism $\phi \wedge \phi : E \to E \otimes \Omega_X^2$ vanishes. A Higgs bundle is a locally-free Higgs sheaf. A subsheaf F of E is called the Higgs subsheaf if $\phi(F) \subset F \otimes \Omega_X^1$, i.e. the pair $F = (F, \phi|_F)$ becomes itself a Higgs sheaf. Let (E_1, ϕ_1) and (E_2, ϕ_2) be two Higgs sheaves over X. A morphism between them is a map $E_1 \to E_2$ such that the following diagram commutes

$$\begin{array}{ccc} E_1 & \xrightarrow{\phi_1} & E_1 \otimes \Omega_X^1 \\ f\downarrow & & f\otimes 1\downarrow \\ E_2 & \xrightarrow{\phi_2} & E_2 \otimes \Omega_X^1. \end{array}$$