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Abstract

We provide a geometric method to stabilize asymptotically with phase an arbitrary fixed periodic orbit of a locally generic three-dimensional Hamiltonian dynamical system. The main advantage of this method is that one needs not to know a parameterization of the orbit to be stabilized, but only the values of the Hamiltonian and a fixed Casimir (of the Poisson configuration manifold) at that orbit. The stabilization procedure is illustrated in the case of the Rikitake model of geomagnetic reversal.

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Keywords: Hamiltonian dynamics; periodic orbits; asymptotic stability with phase.

1 Introduction

The purpose of this article is to provide a method to stabilize asymptotically with phase an arbitrary fixed periodic orbit of a locally generic three-dimensional Hamiltonian dynamical system. The main advantage of this method is that one needs not to know a parameterization of the orbit to be stabilized, but only the values of the Hamiltonian and a fixed Casimir (of the Poisson configuration manifold) at that orbit. Moreover, if there are many periodic orbits located on the same common level set of the Hamiltonian and the Casimir, then the same perturbation can be used in order to asymptotically stabilize all of them in the same time. The method can be applied for a large number of concrete dynamical systems coming from various sciences, which admit three-dimensional Hamiltonian realizations, e.g., Euler's equations of free rigid body dynamics ([5], [2]), the Rikitake system([11]), the Rössler system ([13]), the Rabinovich system ([14]), etc.

Before explaining how the stabilization method works, let us clarify what is meant by a locally generic three-dimensional Hamiltonian system. Formally, a *locally generic three-dimensional Hamiltonian system* is a three-dimensional Hamiltonian system restricted to an open neighborhood around a regular point of the Poisson configuration manifold. Recall that in contrast to Casimir invariants (which globally may not exist), locally, around each regular point, every Poisson manifold admits *local Casimir invariants* (i.e., Casimir invariants of the restricted Poisson structure). Accordingly, a locally