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Crystalline Cohomology of Superschemes

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Abstract

We introduce a notion of crystalline cohomology for superschemes and show that it is isomorphic to the usual crystalline cohomology of the underlying commutative scheme if 2 is invertible on the base.

Wess-Zumino terms play a crucial role in the matching of theoretical predictions in the framework of Quantum Chromodynamics of the decay of the neutral pion π^0 into two photons

$$\pi^0 \longrightarrow \gamma + \gamma$$

and the experimentally observed frequency. Wess-Zumino terms are often known to correspond to suitable cohomology groups related to de Rham cohomology of the target space of a suitable Σ-model. There are situations of physical interest where the target space is not a commutative manifold but rather a superspace. For example, consider the Green-Schwarz Wess-Zumino term in the superstring action as discussed in [10]. This concerns a Σ-model quantum field theory with maps

$$\phi : \Sigma \longrightarrow M^{\mathrm{super}}$$

where Σ is a surface and M^{super} is a certain supermanifold whose associated commutative manifold is simply $\mathbb{R}^m$ for a suitable m. The Wess-Zumino term is given by the integral

$$\mathrm{WZ}[\phi] := \int_T \Omega_3$$

where T is a subspace of M^{super} with boundary $\phi(\Sigma)$ and Ω_3 is the restriction to T of a suitable 3-form on M^{super}. If one wants a cohomological interpretation of this Wess-Zumino terms it is useful to have comparison theorems between the cohomology of superspaces and the cohomology of the associated commutative spaces. For the Green-Schwarz superstring the work of Kostant [6] is sufficient, it shows that the suitably defined de Rham cohomology of a supermanifold agrees with the usual de Rham cohomology of the underlying commutative, or "bosonic", manifold. In this work, a more arithmetic version of such a cohomological comparison theorem will be shown. One motivation for this is the following:

One can conjecture that developing tools for quantum field theories over more general ground rings than the real or complex numbers could be useful. An example of this is the work [7] of Kontsevich-Schwarz-Vologodsky that approaches the integrality question of instanton numbers via a p-adic version of the B-model topological string. This is a motivation for us to study cohomology theories

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